## Math: Fractions and Decimals 105

Many students face fractions with trepidation; "they're too hard, I don't understand". If this is you, there is no better tool to bring yourself back up to speed than a tape measure. A tape measure is a number line. Addition is moving forward and subtraction is moving back. Get a tape measure and we'll look at it together.

Look at how an inch is divided up. The one pictured has the inch divided in 16 pieces.

## Understand what $1 / 16$ is.

 1/16 (one-sixteenth) of an inch is usually the smallest measurement on a tape measure. The distance between every line on the tape measure is $1 / 16$ of an inch.Understand what 1/8 is. $1 / 8$ (one-eighth) of an inch is twice as big as $1 / 16$ of an inch. It is every other mark. Notice we have dotted every other one. $1 / 8$ is twice as big as $1 / 16$.
Understand what $\mathbf{1 / 4}$ is. 1/4 (one-quarter) of an inch is twice as big a $1 / 8$ of an inch. It is every fourth mark. Also note $1 / 4$ is 4 times as

big as $1 / 16$.

## Understand what $1 / 2$ is.

$1 / 2$ (one-half) of an inch is twice as big as $1 / 4$. It is four times as big as $1 / 8$ and eight times as big as $1 / 16$.
Understand what an inch is. The large markings on the tape measure are inches. They are numbered to proceed (from the left) the mark. An inch is twice as big as $1 / 2,4$ times as big as $1 / 4,8$ times as big as an $1 / 8$, and 16 times as big as $1 / 16$. Do you see the pattern?

## Equivalent fractions

$8 / 16=4 / 8=2 / 4=1 / 2$

## Try it



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1" = $\qquad$ $12=$ $\qquad$ $14=$ $\qquad$ $18=$ $\qquad$
$3 / 4^{\prime \prime}=6 /$ $\qquad$ = 12/

## Adding and Subtracting

The picture describes simple addition. Where the numbers cross over, the end of the tape provides the answer.

For example: $43+5=$ end of tape 48.
$44+4=$ end of tape $48.45+3=$ end of tape 48 and so on.

This is actually a very common practice. You nail boards together. You weld pieces and are asked "how long is the result of your work?"

But everything you measure won't be exact to the inch in length. Pick up anything, most likely if you read the tape as closely as you can, you will end up with a fraction.

Warning: Do not to permanently bend the tape measure by bending too sharply.

Try something a little more challenging: 39-5/8 + 6-9/16 = $46-3 / 16$. Play with this technique in subtraction. You can become quite proficient in a short while.


Within the inch: $3 / 8+3 / 8=6 / 8$ or $3 / 4$, right? Use $60^{\prime \prime}$ as the starting point. Go to $60-3 / 8$. Put the lower tape on any start point. We're using 13 in this photo. Go over $3 / 8$ towards the 12, now 'cross' to the top for the answer: $3 / 4$. No long hand needed when you are familiar with a tape.

Going back to the idea of $60^{\prime \prime}$ as the starting point, having a number line handy makes introduction of negative numbers easy. What if we just call $60^{\prime \prime}$ as Zero? Move right from 60 to add and move left to subtract.

The tape measure will also allow you to introduce mixed numbers quickly into the classroom and also to frame the discussion of adding unlike fractions and equivalent fractions.

How to begin? Get a pile of scraps of anything lying about, such as $2 \times 4$ pieces or chunks of iron. Label them A, B, C, D etc. and measure several times until you get a consistent answer every time. Try to guess lengths before you measure. You should be able to guess down to 1" and measure with precision and agreement down to $1 / 16^{\prime \prime}$.


Image supplied by author

Now take your pieces and answer.

What is $A+C ?$
What is D-A ?
Addition with a Tape
To add with a tape, we mark the first number, move our tape to that mark, and measure off the second number in the same direction. Addition when we have only feet, inches, or fractions is just addition. It becomes much more interesting when we are measuring and end with a mixed number which could be in feet, inches and a fraction. Handle the parts separately, beginning with the largest unit. You may need to clean up the answer. Example:

$$
\begin{array}{r}
5^{\prime} 6^{\prime \prime} \\
+9^{\prime} 3^{\prime \prime} \\
\hline 14^{\prime} 9^{\prime \prime}
\end{array}
$$

Simple enough, but what happens when you have more inches than there are in a foot?

$$
\begin{array}{r}
5^{\prime} 6^{\prime \prime} \\
+9^{\prime} 11^{\prime \prime} \\
\hline 14^{\prime} 17^{\prime \prime}
\end{array}
$$

In this case, we need to carry across the units to convert the answer from $14^{\prime} 17^{\prime \prime}$ to $15^{\prime} 5^{\prime \prime}$. The same thing can happen with a fraction.

3' 7-1/2"
+2'5-3/4"
$5^{\prime} 12-5 / 4^{\prime \prime}$ This answer would convert to: 6' 1-1/4"

## Carry and Borrow

When we first learned to add, we learned about units: ones, tens, hundreds, thousands, and so forth.

In arithmetic, whenever we have a sum greater than 9 in any column, we will need to carry over into the next column. Example:

$$
\begin{array}{r}
17 \\
+\quad 14 \\
\hline=31
\end{array}
$$

In order to get the answer (31), we had to carry the one after we added $7+4$. The sum from the "ones" column added up to 11 . We leave the 1 from the "ones" column and carry the 1 from the "tens" column over. We then have three 1 s in the "tens" column which adds up to 3.

When using a tape measure, this comes up often. $12^{\prime \prime}=$ $1^{\prime}$, so $13^{\prime \prime}$ is the same as $1^{\prime} 1^{\prime \prime}$.

A fraction of an inch works the same way. 8/16 + 9/16 = $17 / 16$, which is the same as $1-1 / 16^{\prime \prime}$.

When the fractions do not share the same denominators (the bottom number in the fraction), we make the problem easier if we convert them to have a common denominator.

## EXAMPLE:

Add $1 / 2$ and $1 / 4$. First convert $1 / 2$ to share a common denominator with $1 / 4.1 / 2=2 / 4$, so simply add $2 / 4$ and $1 / 4$ to get $3 / 4$.

## Subtraction with a Tape

Now we are ready to look at subtraction with the tape measure. Subtraction is the opposite of addition, moving left on the number line (tape measure). Mark off the first value and move the tape to this mark.

When you subtract you measure back to where you started.

## EXAMPLE:

$40-3 / 16-8-1 / 16=32-2 / 16$ or $32-1 / 8$
$32-1 / 8-8-1 / 8=24{ }^{\prime \prime}$

## Try it

12
$-31 / 8$

## This is the same as if we had written:

11-8/8
$-31 / 8$

Solve the fraction first: $8 / 8-1 / 8=7 / 8$
Then, complete the problem by subtracting the whole numbers: 11-3=8

The answer is $8-7 / 8$.
Just as in addition, sometimes the problem is mixed. To make it easier, we need to rewrite it using easier terms like this:

35-1/4
-27-1/8

Remember to use a common denominator for addition and subtraction. The problem can be rewritten like this:

35-2/8
$\begin{array}{r}-27-1 / 8 \\ \hline 8-1 / 8\end{array}$

## Multiply with the Tape

You are asked for three boards measuring 3' 2" each. How many board feet are needed? Use the tape to repeatedly add by marking off the first, moving the tape to the mark and marking off the second, and then move the tape to the second mark and measure the third $3^{\prime} 2^{\prime \prime}$. You may see this as time consuming. Is there a better way? Handle the feet first, then the inches, then the overflow.
$3^{\prime} \quad 2^{\prime \prime} \times 3$
$3^{\prime} 2^{\prime \prime}+3^{\prime} 2^{\prime \prime}+3^{\prime} 2^{\prime \prime}=9^{\prime} 6^{\prime \prime}$
$3^{\prime} 8^{\prime \prime} \times 3=9^{\prime} 24^{\prime \prime}=11^{\prime}$

What about multiplying with a fraction?
What is $1 / 2$ of $12^{\prime \prime} ? 1 / 2$ of $10^{\prime \prime} ? 1 / 2$ of $13^{\prime \prime}$ ?
What is $1 / 2$ of $1 / 2^{\prime \prime}$ ? $1 / 2$ of $1 / 4^{\prime \prime}$ ? $1 / 2$ of $1 / 8^{\prime \prime}$ ? Do you see the pattern?

What is $1 / 2$ of 4"?
Is this division or multiplication? How are they similar? Dividing by 2 is the same as multiplying by $1 / 2$. Dividing by 5 is the same as $\qquad$ ?

What is $1 / 2$ of $1 / 8$ ?
What is $1 / 2$ of $4^{\prime} 10^{\prime \prime} ? 1 / 2$ of $6^{\prime} 6^{\prime \prime}$ ?
Measure a $2 \times 4$. How much would be $1 / 4$ of it?

## What about a mixed problem?

$4-1 / 8 \times 12-1 / 4=(4-1 / 8 \times 12)+(4-1 / 8 \times 1 / 4)$
Hint: Remember to borrow what you need.

## Division with a Tape

What is $1 / 2$ of $1 / 8$ ?
How many boards, each $3^{\prime} 3^{\prime \prime}$, can I cut from an $8^{\prime}$ board?
From a $10^{\prime}$ board?
How many quarters are in an inch?
How many eighths are in an inch?
How many eighths are in two inches?
Looking at your measuring tape, this is easy.

## Division is the opposite of multiplication.

Always invert the second number.
Works with whole numbers as well $8 \div 5=8 \div 5 / 1=8 \times 1 / 5$
What is $1 / 2$ of $8 / 8$ ? $8 / 16$.
What is $1 / 2$ of $16 / 16$ ? $16 / 32$.
Remember: $1 / 2$ is the same as $2 / 4$ is the same as $4 / 8$ is the same as $8 / 16$ is the same as $16 / 32$.

Idea for thought: Some may ask what about all the other fractions, like $12 / 61$ and $7 / 53$ etc? What if your students knew just whole inches, halves, quarters, eighths, sixteenths and 32nds and 64ths real well?

Try these (with or without your tape):
$3-5 / 16^{\prime \prime}+2-3 / 16^{\prime \prime}=\quad$ convert
$3-5 / 16^{\prime \prime}-2-3 / 16^{\prime \prime}=\quad$ convert
$5-1 / 8^{\prime \prime}+3-7 / 8^{\prime \prime}+4-3 / 8^{\prime \prime}=$ $\qquad$

## Now do this with a number line. It's not a ruler, so no dimensions.



## What fractional piece of the total line is each?

A =
D =
$B=$
E =
$\mathrm{C}=$
$\mathrm{F}=$

