UNIT 5
INSTRUCTOR'S MANUAL

BOOLEAN ALGEBRA

1. This unit presents the logic theory which underlies PLC programming. It is probably the most abstract, and therefore the most difficult, of the background units. If the students have serious difficulty with this information, the unit can be omitted. However, you must then be careful to work around later instructional materials which make reference to Boolean Algebra.

2. The Experiment section requires the student to build circuits which model the Boolean relationships. It may be necessary to help the students trace the paths of the circuits to understand their AND and OR elements.

3. Questions 5 and 6 in the Questions section require the students to create Boolean equations for two different logical situations. If the students need to, these questions could be answered in a group activity. For both questions, it will help the students if they list each of the conditions in the logical relationship before trying to develop the Boolean equation

UNIT 5

BOOLEAN ALGEBRA

Objectives

Upon completion of this unit the student will be able to:

1. Explain what Boolean Algebra is.
2. Explain the AND, OR and NOT logic functions.
3. Develop simple logic equations.
4. Equate a simple electrical circuit to a simple logic equation.

Background

In 1874 George Boole developed basic laws and rules for a form of mathematics that could be applied to logic problems. It became known as *Boolean Algebra*, and its applications have grown in the years since its development. In 1938 Claude Shannon, a scientist for Bell Laboratories, used Boolean Algebra for analyzing the switching networks in the telephone system. Boolean Algebra has since been applied to computers and PLCs.

Boolean Algebra provides a mathematical model for a *logical decision*. A logical decision is a decision which can have only one of two possible outcomes, YES or NO. Other equivalent outcomes are TRUE or NOT FALSE, and FALSE or NOT TRUE. The Boolean model uses "1" to represent YES and "0" to represent NO. Equivalents of YES and NO are given in Table 5-1.

YES = TRUE = NOT FALSE = 1

NO = FALSE = NOT TRUE = 0

Table 5-1
Equivalents of YES and NO

Every logical problem has one or more *conditions* which must be met. The decision will be TRUE if the conditions are all met, and FALSE if any one condition is not met. For example, there are two conditions which must be met in order for a traditional clock to indicate 5:00. First, the large hand must be on the 12; second, the small hand must be on the 5. If both conditions are TRUE, then the result is TRUE. It is 5:00. If either condition is FALSE, then the result is FALSE. It is not 5:00.

Boolean Symbols

For a more simple statement of the example above, and problems like it, Boolean Algebra uses several symbols. These symbols are applied to letter which are used to represent the conditions, or variables, in a logical problem. The symbols allow the verbal explanation of the problem to be replaced by a mathematical equation. A *truth table* shows the relationships among a given set of variables.

Inverse Operation - NOT Function. The *logical inverse* operation, or *NOT* function, changes a logical TRUE (1) to a logical FALSE (0) or the reverse. The symbol for the inverse operation is a horizontal line above the letter representing the variable.

Thus, if $\overline{A}$ is the variable, A is its inverse and is read "A-NOT" or "A-INVERSE." The truth table for A-INVERSE is given in Table 5-2.

A	$\overline{A}$
0	1
1	0

If A is 0, then $\overline{A}$ is 1.
If A is 1, then $\overline{A}$ is 0

Table 5-2
Inverse Operation Truth Table

Logical AND Operation - AND Function. The *AND* function requires that any two conditions or variables connected by AND must both be TRUE for the decision to be true. The example of a clock reading 5:00 used the AND function.

AND is symbolized by a dot between the symbols for the variables. So, if A and B are two variables, when A is "ANDed" to B, it is represented A.B. Table 5-3 is the truth table for A.B.

A	B	A.B
0	0	0
0	1	0
1	0	0
1	1	1

Table 5-3
Logical AND Truth Table

Switches in an electrical circuit can be placed in an AND relationship, such as that pictured in Figure 5-1. For the light to be turned on, both switches must be closed. Any other condition results in no flow of electric current, so the light remains off.

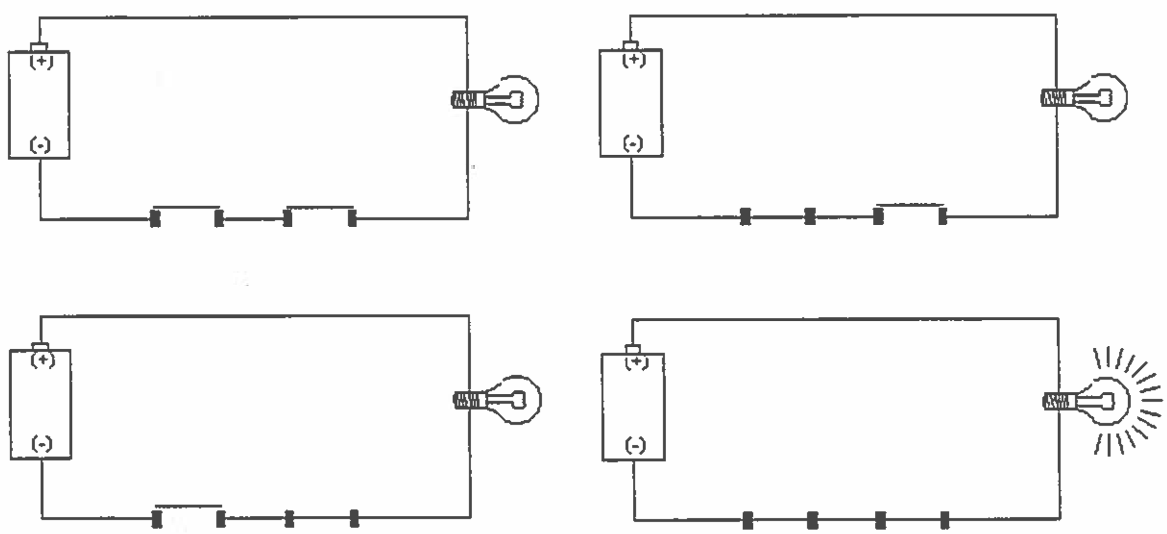


Figure 5-1
ANDed Switches

<u>Logical OR Operation - OR Function</u>. With the *OR* function, it is sufficient that any <u>one</u> condition be TRUE for the logical decision to be TRUE. The OR function is symbolized by a plus sign (+) placed between the variables, so condition A OR condition B looks like A+B. The OR truth table is shown in Table 5-4.

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

Table 5-4
Logical OR Truth Table

The OR function can be used in the construction of electrical circuits, as shown in Figure 5-2. In these circuits, it is sufficient for one switch to be closed for the light to come on.

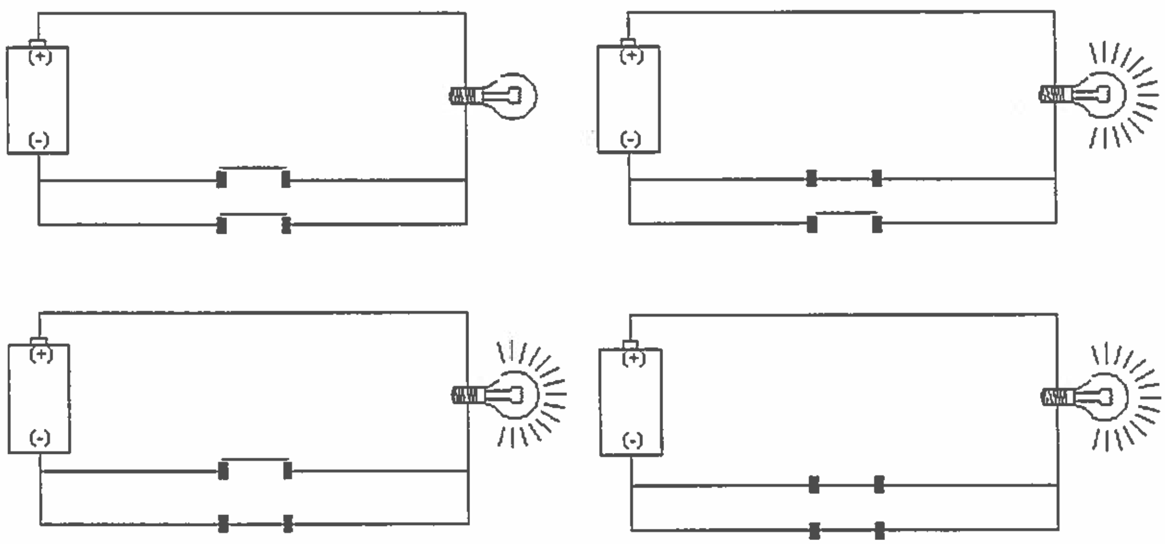


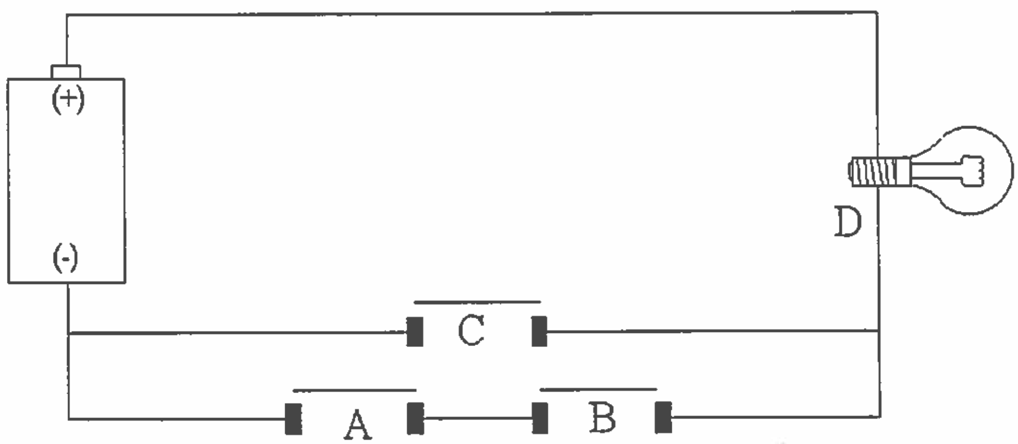
Figure 5-2
ORed Switches

AND, OR and NOT functions can be combined in a variety of ways to reach logical decisions. In an example concerning the weather, if it is snowing AND NOT windy, OR if it is sunny, it is reasonable to go outdoors. In Boolean form,

If A = snowing
B = wind
C = sun
D = go outdoors

Then $(A \,.\, \overline{B}) + C = D$

Figure 5-3 offers a second example, using AND and OR in an electrical circuit.



A	B	C	(A.B) + C
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Figure 5-3
Electrical Circuit

Switches A and B are ANDed together and then ORed with switch C. The Boolean equation for this circuit is

$$(A \,.\, B) + C = D$$

where D represents the light. The equation says that if switches A AND B are closed, OR if switch C is closed, the light D will come on. The truth table is given with the illustration in Figure 5-3.

Note that the parentheses ( ) around A.B require you to take these variables together -- A and B are <u>both</u> closed. The equation

$$A . (B + C) = D$$

then means that when switch A is closed AND either switch B OR switch C is closed, light D will come on. If your were to draw the electrical circuit, it would be quite different from the circuit in Figure 5-3. Its truth table would also be different.

Examine the circuits in Figures 5-1, 5-2 and 5-3. In Figure 5-1, the ANDed switches are connected in a series circuit. The ORed switches in Figure 5-2 create a parallel circuit. AND/OR combinations as in Figure 5-3 are partly parallel and partly series circuits. If you need help understanding this, see Unit 5 for a refresher on basic circuitry.

EXPERIMENT

Purpose

To represent Boolean equations by electrical circuits.

Procedure

1. Locate your jump wires and independent switch and light.

2. Create the circuit shown in Figure 5-4 to represent the equation:

$$A.B = C$$

where A and B are switches and C is a light bulb.

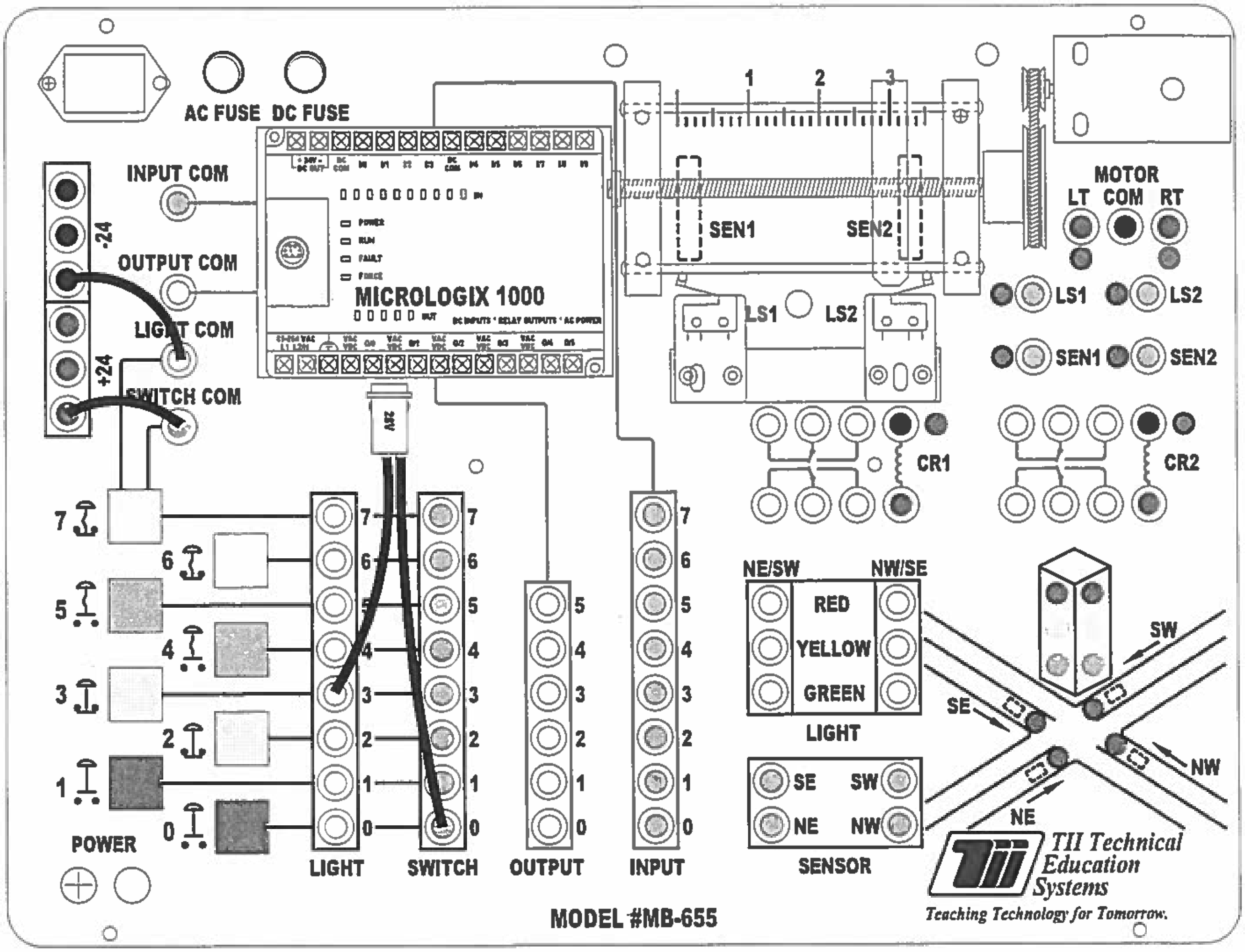


Figure 5-4
Circuit for Step 2

3. Turn on the power.

   a. What happens to the lights when you depress only switch 0?

      Nothing.

   b. What happens to the lights when you depress only the independent switch?

      Nothing.

4. Experiment with the switches.

   a. What must you do to make a light come on? Which light was it?

      Switch 0 and the independent switch must be pressed simultaneously in order for light 3 to come on.

b. Describe how this circuit represents the equation A.B=C.

Switch 0 AND the independent switch must be on (True) in order for light 3 to be on (True).

5. Disassemble the circuit.

6. Construct circuit like the one shown in Figure 5-5. It will represent the equation

$$A + B + C = D$$

where A, B and C are switches and D is a light.

You may use switches 0, 1, 4, and 5 only and any light you wish.

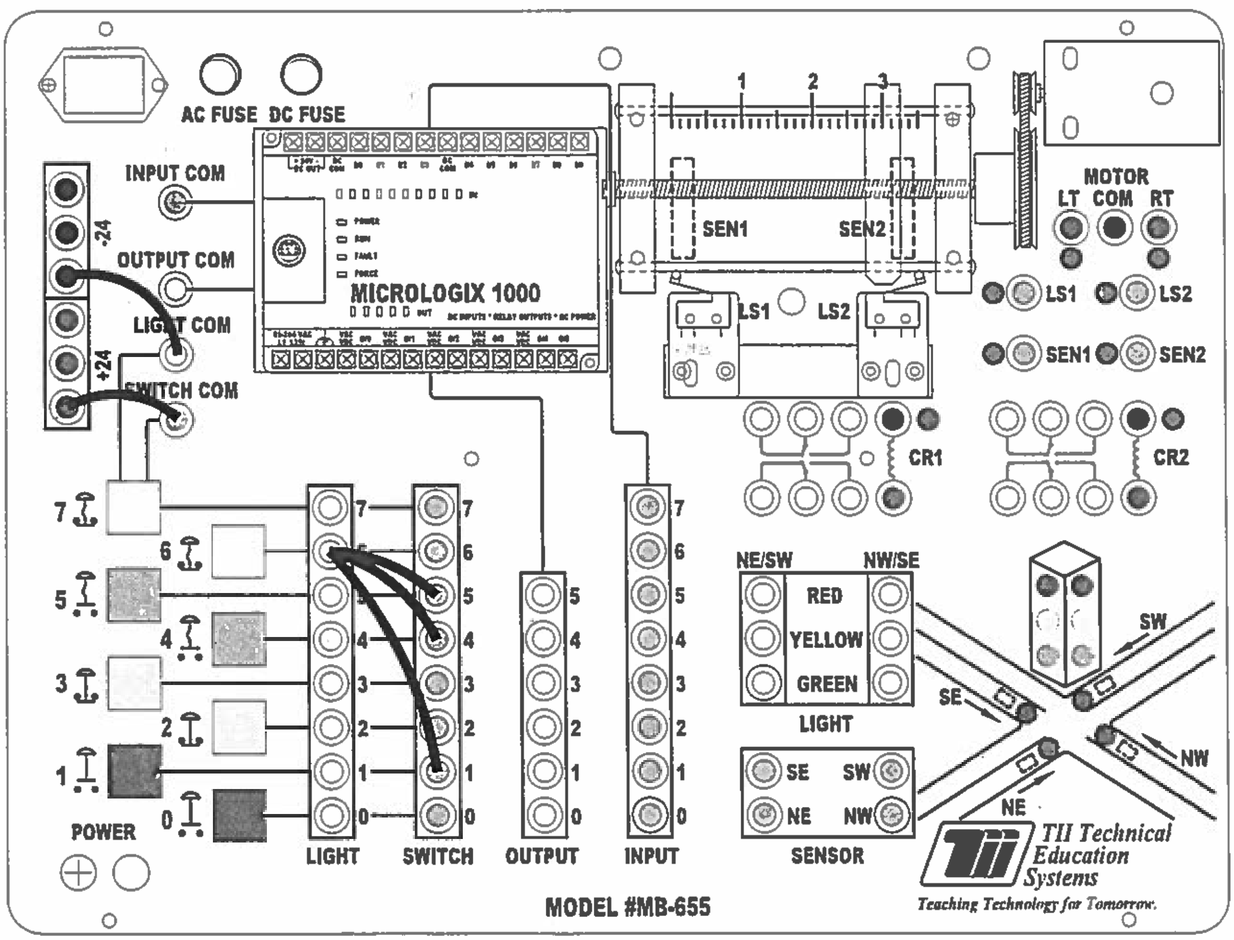


Figure 5-5
Circuit for Step 6

7. Test the switches.

a. What must you do to get the light to come on?

Any one switch must be pressed (on). If it is a momentary switch (0 or 1) it must be held down to keep the light on.

b. How does this circuit represent A+B+C=D?

Only one of the three switches must be on (True) for the light to come on (True). In the equation, only one of the three variables A, B, or C must be true for D to be true.

8. Disassemble the existing circuit.

9. Try to construct the circuit described by

$$(A + B) . C = D + E$$

where A, B, and C are switches and D and E are lights.

Choose from switches 0, 1, 4, 5 and the independent switch. Use any lights you wish.

Remember: ( ) means you must take these variables together.

a. Put this equation into words.

Either A or B must be True, and C must be True, in order for D or E to be True.

b. Draw a picture of the circuit on Figure 5-6. Identify the input and output connectors you will use.

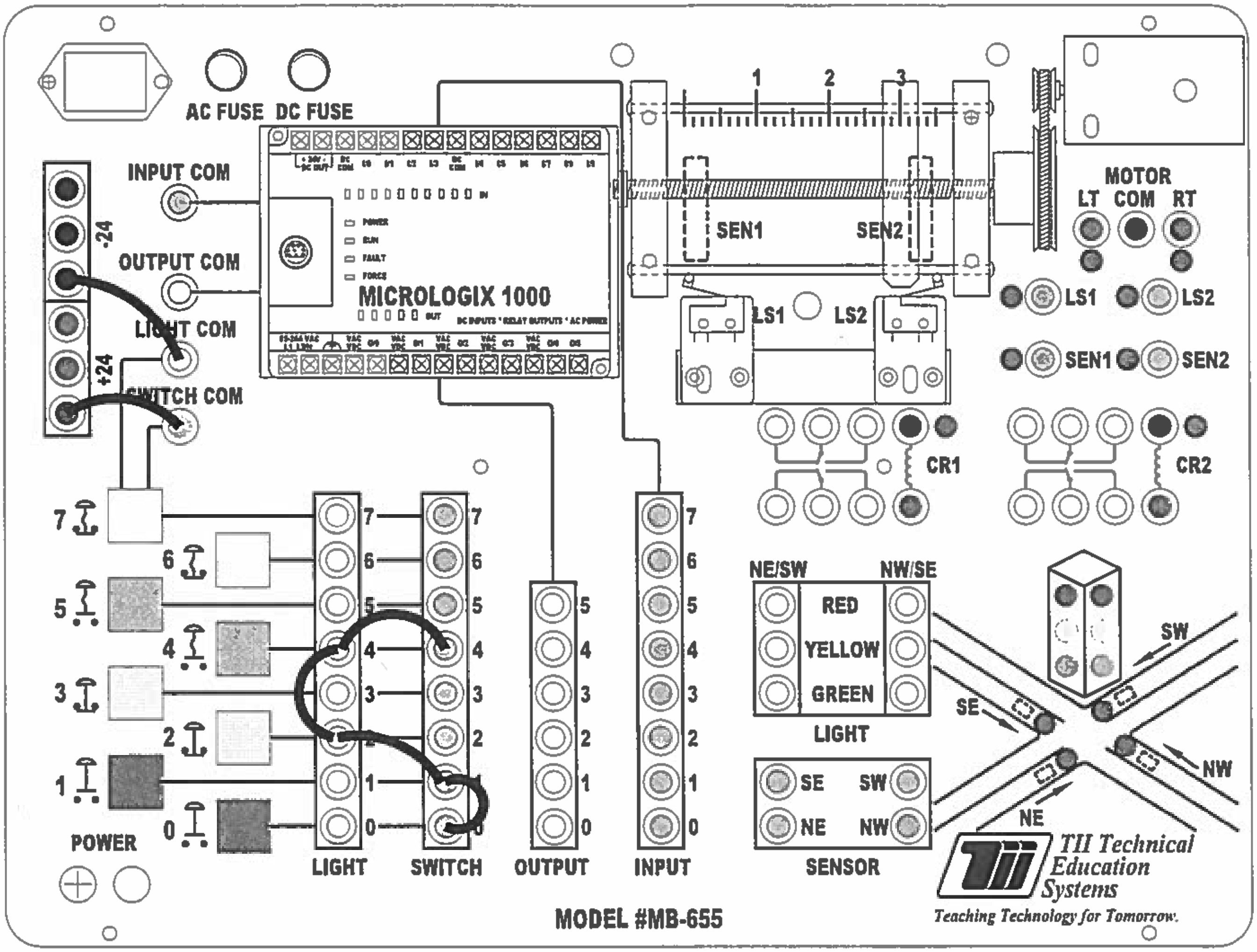


Figure 5-6
Circuit for Step 9

Inputs: Any two (2) of the switches 0,1,4, and 5 must be connected to the independent switch.

Outputs: The independent switch must be connected to any two (2) lights, 0-7.

The independent switch functions in series with either of the other two switches.

10. Try to light the bulb.

a. What did you have to do?

The independent switch and one of switches 0,1,4,5 (whichever was selected) must be pressed simultaneously for the lights to come on.

b. Does your circuit properly represent the equation?

Answers will vary.

c. How can you tell it does (does not)?

To test the circuit against the equation, test every possible combination of the four switches (0,1,4,5). Identify those switches, or combinations of switches, which turn on the two lights, and compare to the given equation.

11. If your circuit was incorrect, study Figures 5-4 and 5-5 for help. Now try again.

12. Think about the two types of switches available on the panel.

a. When might the NOT function be used to represent one of the available inputs?

The NOT function could be used when a normally closed switch must not be pressed to complete a circuit.

b. Construct a simple circuit using a normally closed switch. Draw the circuit on the panel and write its Boolean equation below.

An example of a simple use of the NOT function is shown below. The student's answer may be different but should employ the same logic.

$$A \cdot \bar{B} = C$$

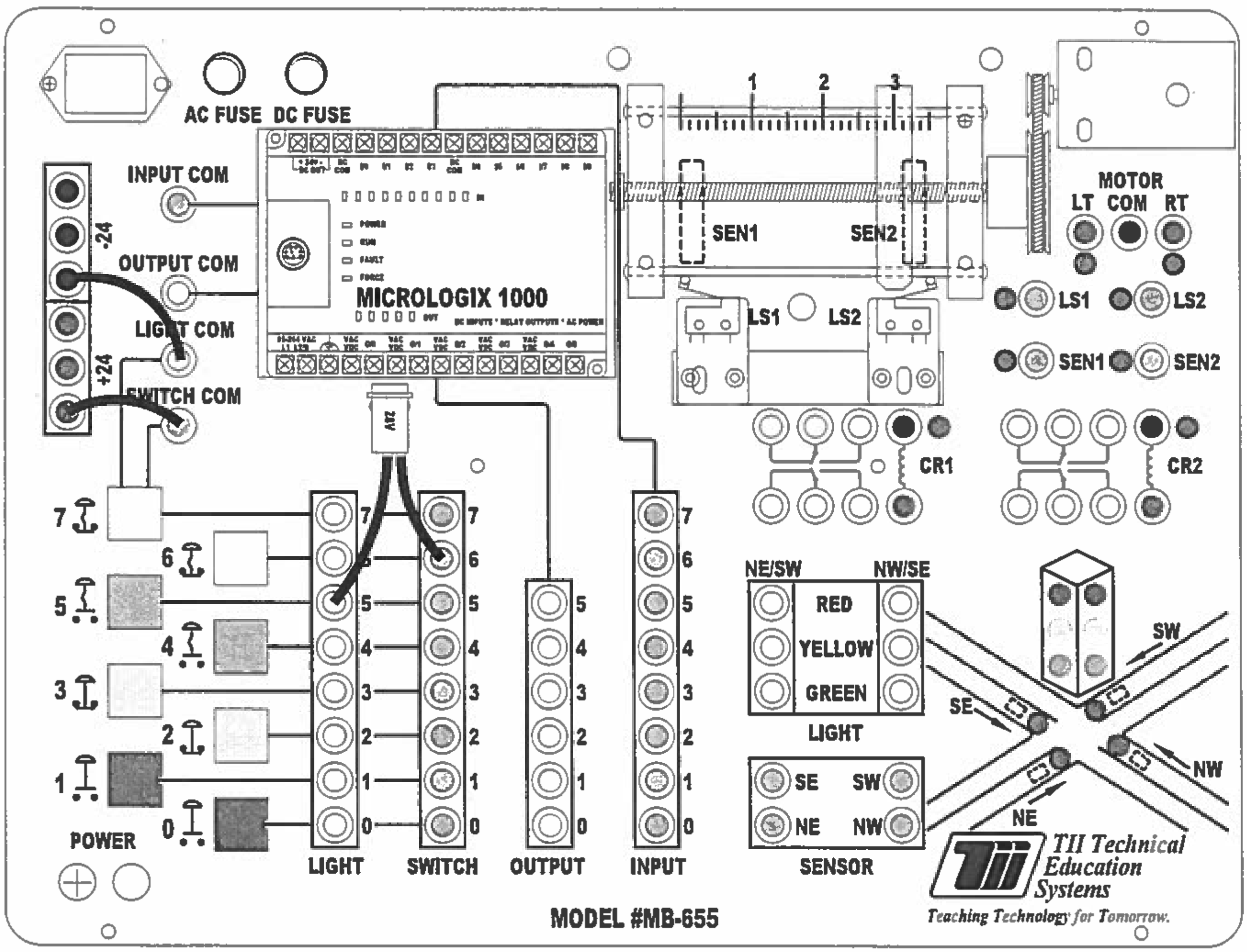


Figure 5-7
Circuit for Step 12

13. Compare your answers in the Experiment section to those provided at the end of the unit. When you are satisfied that you understand the materials, set aside the kit and complete the Questions which follow.

Questions

1. What is the purpose of Boolean Algebra?

   Boolean Algebra uses mathematical equations to represent logical decisions and logical relationships. It includes AND, OR and NOT functions, represented by symbols in the equations.

2. Explain how a Boolean equation represents a logical decision. Use an example if this will help.

   A logical decision is a decision which can only be YES/NO or TRUE/FALSE. The decision is reached by testing conditions which relate to the decision. All conditions must be TRUE for the decision to be TRUE. If any one condition is FALSE the decision must be FALSE. Boolean Algebra uses letters to represent each condition and symbols to relate one condition to the other. The letters and symbols are then combined into an equation which states the conditions and their logical relationships. The truth table for the equation lists every possible combination of true and false and the decision for each combination.

3. How is an electrical circuit like a Boolean equation?

   OR is equivalent to a parallel circuit. AND is a series circuit. NOT changes the status of the input (from on to off or from off to on).

4. Define:

   a. A . B *A and B must both be true for a true outcome.*

b. A + B *Either A or B must be true for a true outcome.*

c. A *A-NOT or inverse of A. When A is true, A is false and vise versa.*

5. What is the Boolean equation for the following:

If the ticket prices are not too high and I have enough money, or if you help pay for the tickets, we can go to the show.

Be sure to show what each variable represents.

A = tickets not too high

B = enough money

C = you help pay

D = we go to show

(A . B) + C = D

Note: if A = tickets too high, then the equation would be: (A . B) + C = D

6. What is the Boolean equation for the circuit in Figure 5-8?

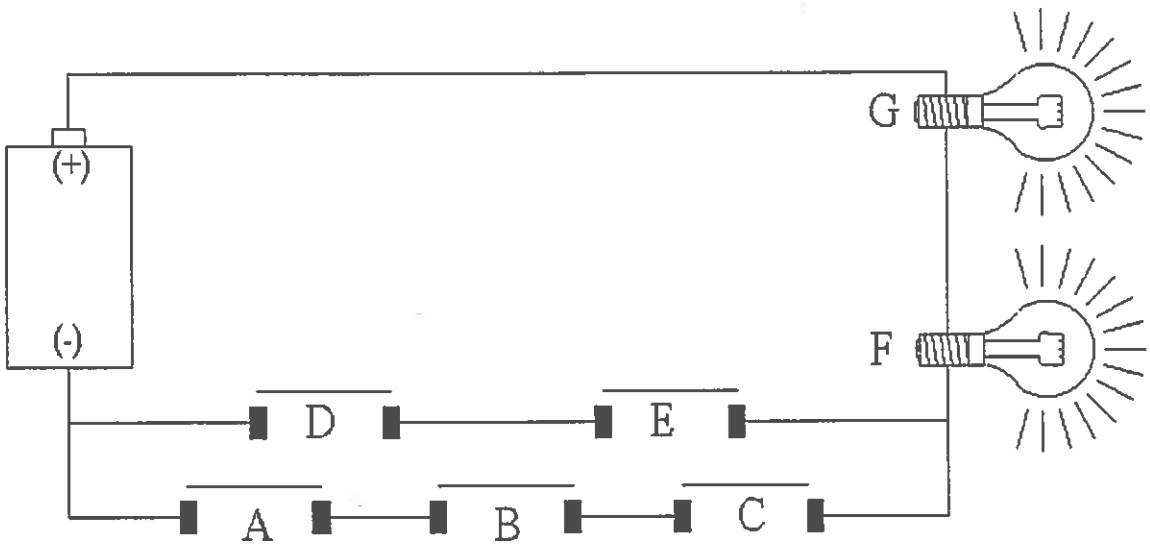


Figure 5-8

(A . B . C) + (D . E) = F . G

UNIT 6
INSTRUCTOR'S MANUAL

LADDER LOGIC DIAGRAMS

1. This unit provides information on the basics of PLC programming. It should be covered very carefully and thoroughly to ensure that the students master the materials. Virtually all units which follow will use ladder logic diagrams in some capacity.

2. The Experiment section of this unit presents several problems in ladder logic diagramming. An Answer Key for the Experiment problems follows at the end of the unit. Use this to provide immediate feedback to the students on their comprehension of ladder logic diagramming.

3. In order to confirm a student's knowledge of the basics of ladder logic diagramming, check the answer to Question 6 in the Questions section.

UNIT 6

LADDER LOGIC DIAGRAMS

Objectives

Upon completion of this unit the trainee will be able to:

1. Describe a ladder rung, ladder rail, and ladder branch.
2. Identify the elements of a ladder logic diagram.
3. Identify and label the steps in a ladder logic diagram.
4. Convert simple circuits into ladder logic diagrams.
5. Convert simple ladder logic diagrams into circuits.

Background

A ladder logic diagram shows all of the possible conditions that exist in an electrical circuit. It is used for describing a circuit to the computer in a PLC and can be thought of as a program. A ladder logic diagram and its equivalent circuit are shown in Figure 6-1 and are described below. Note that every element in the circuit is represented in the ladder logic diagram.

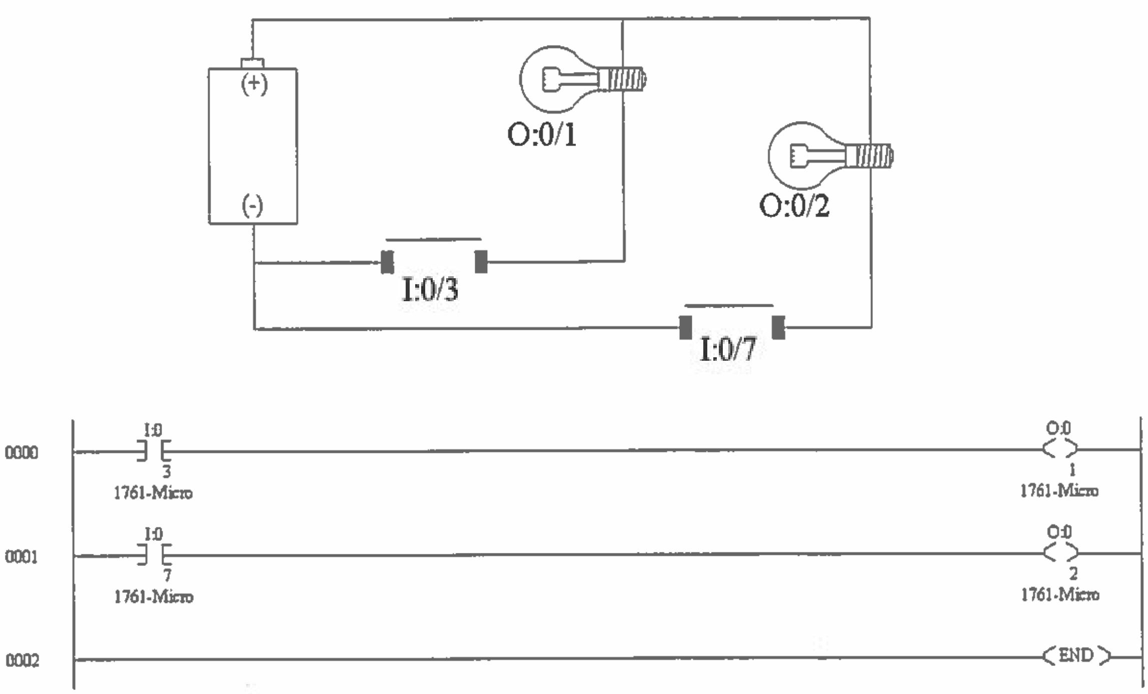


Figure 6-1
Electrical Circuit and Equivalent Ladder Logic Diagram

The ladder logic diagram gets its name from its similarity to a ladder. Each of the horizontal rows on the diagram is called a *rung*. The two vertical lines represent the positive and negative connections to the power source. They are called *rails*. It is standard practice to place the negative (-) or "common" rail on the left and the positive (+) or "hot" rail on the right.

The rungs carry information about the *elements* (inputs and outputs) of the electrical circuit. Although a rung may hold more than one input, each line may hold only one output element, which is shown in the far right end of the line. These outputs are all *coils* of some type. The coils control either output device external to the PLC or they control elements in other rungs of the ladder logic diagram. Any rung may hold one or more input elements. These are always diagrammed to the left of the output element. The ladder logic symbols for input and output elements are given in Table 6-1.

Element	Ladder Logic Symbol
Input	-| |-
Output	-( )-

Table 6-1
Input and Output Symbols

The computer in the PLC must be able to tell the different inputs and outputs apart during programming. To do this, PLC manufacturers have developed ways to *address* each element so the computer knows precisely which element is being entered in a program. The most common types of addresses are sets of specially designated numbers, or combinations of one letter and several numerals. Typically addresses for the Allen-Bradley MiroLogix 1000 PLC's are given in Table 6-2.

Element	Addresses
Input	I:0/0 - I:0/7
Output	O:0/0 - O:0/5

Table 6-2
Input and Output Addresses

Ladder logic diagrams contain other identifying numbers as well. Figure 6-2 is an illustration of a four rung ladder logic diagram. To the left of each rung is a reference number for that rung. In large ladder logic diagrams the reference numbers make it easy to find a specific rung.

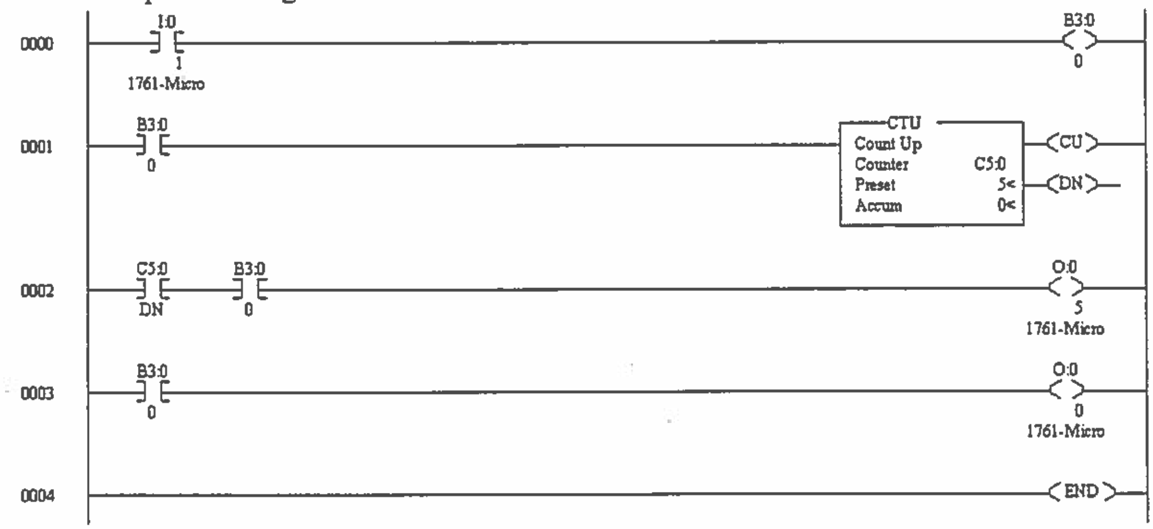


Figure 6-2
Ladder Logic Diagram with Four Rungs

Ladder logic diagrams can be used to represent both series and parallel circuits. The circuit in Figure 6-3 is a simple circuit with two inputs (switches) in series. The circuit can be represented by the ladder logic diagram shown to its right. Input I:0/1 and I:0/2 are connected to switches. I:0/1 and I:0/2 must both be ON for the light connected to O:0/3 to come on. So the two inputs are placed on the same rung of the ladder logic diagram. Remember that the Boolean AND statement can represent this same circuit.

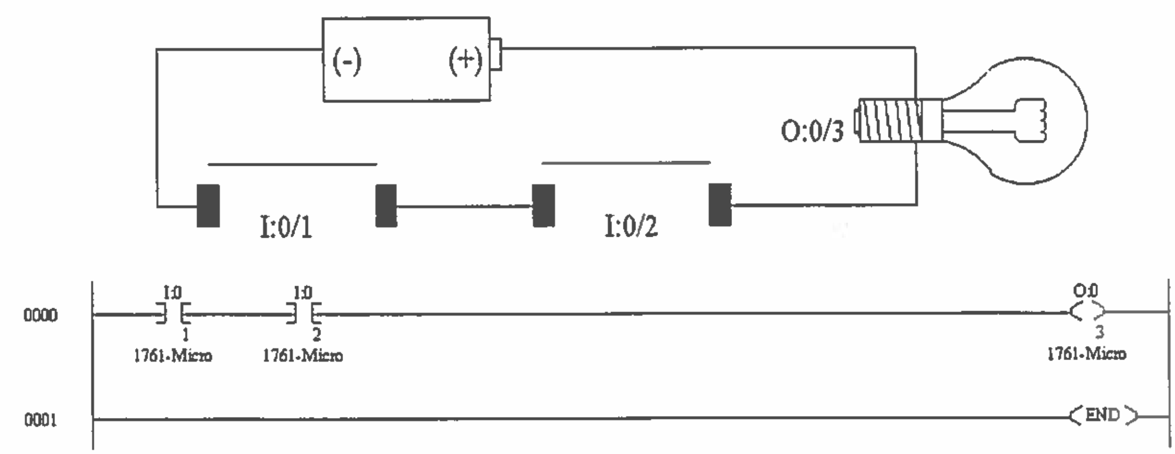


Figure 6-3
Series Inputs and Ladder Logic Equivalent

Outputs are not normally connected in series to a PLC, so there is no equivalent to series outputs in ladder logic diagrams.

Parallel circuits are formed differently in ladder logic diagrams. The circuit shown in Figure 6-4 is a simple circuit which also contains two inputs (switches), this time connected in parallel. The parallel circuit splits to make two separate electrical paths, one for each switch like a Boolean OR equation. Similarly, the ladder logic diagram for this circuit shows two separate paths for the two switches (refer to the logic diagram in Figure 6-4). This type of connection in ladder logic diagrams may contain one or more branches, depending upon the circuit represented. Since branching is very important in ladder logic, it will be covered in detail in a later unit.

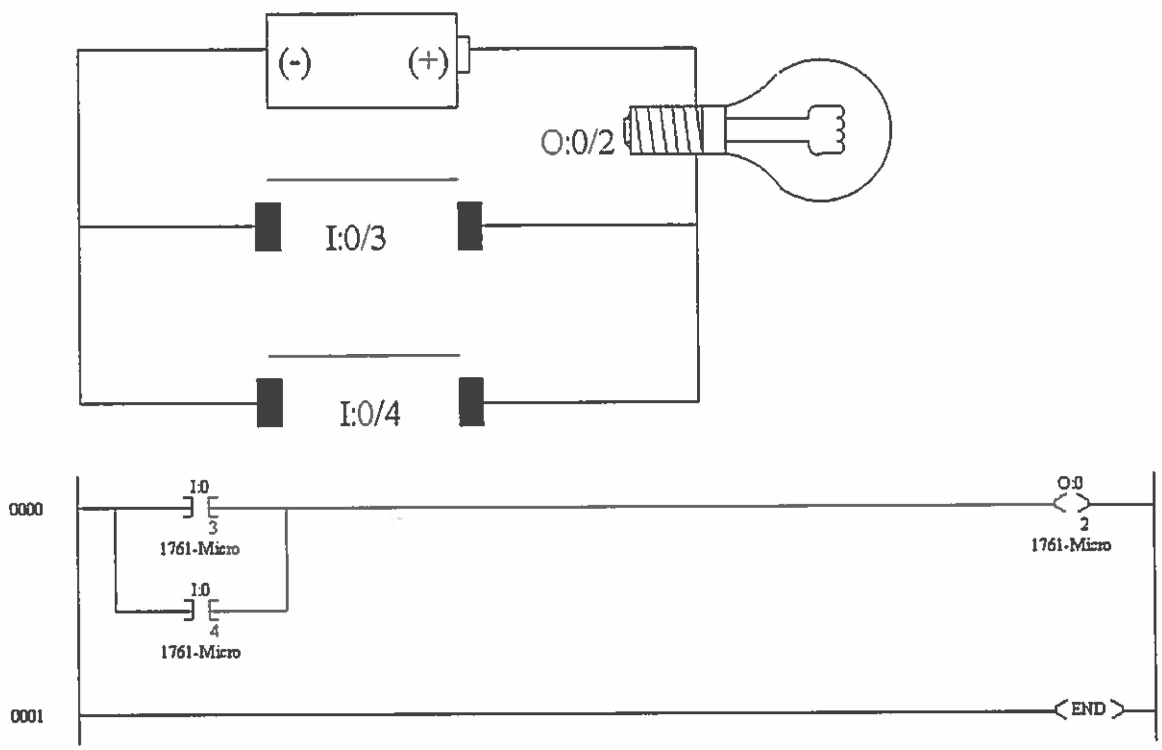


Figure 6-4
Parallel Inputs and Ladder Logic Equivalent

Note that the branch in Figure 6-4 does not have a rung number. The branch is counted as part of the rung to which it is connected.

Another possible parallel circuit has one input (switch) controlling two output coils (lights), as shown in Figure 6-5. The two lights are two different outputs, so each must appear on its own rung of the ladder logic diagram (see ladder logic diagram in Figure 6-5.

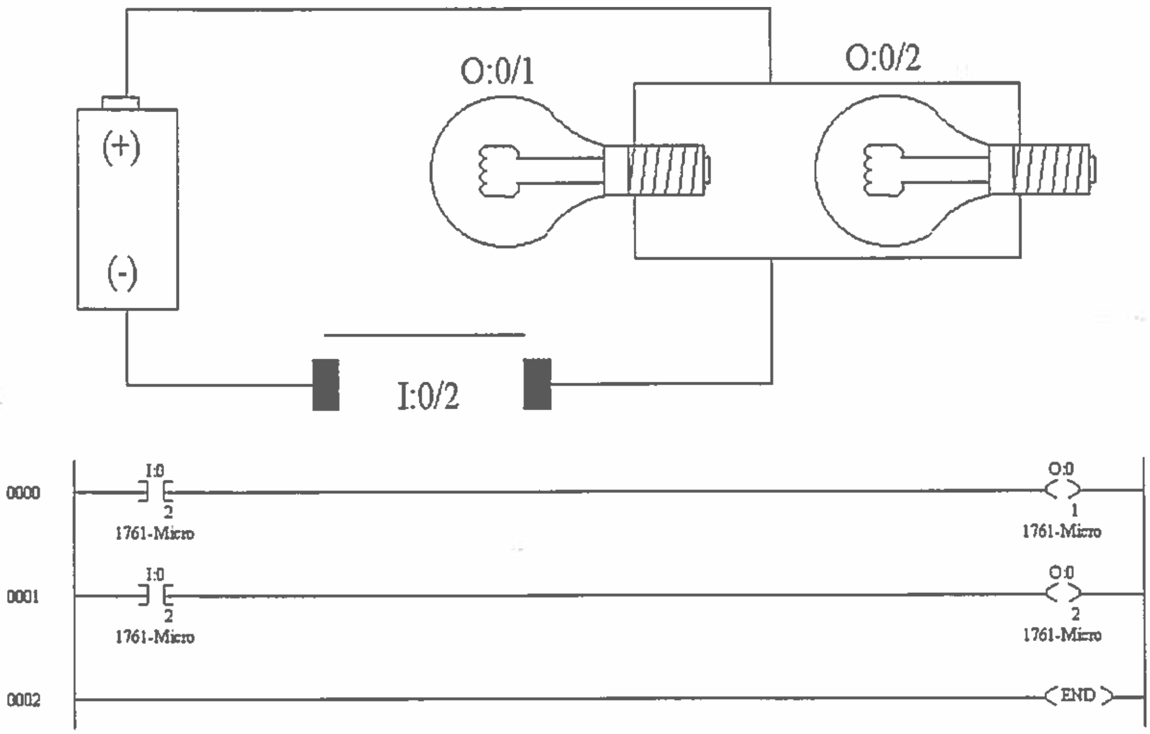


Figure 6-5
Parallel Outputs and Ladder Logic Equivalent

Both the circuit and the ladder logic diagram in Figure 6-5 state that:

1. If switch 2 is ON, light O:0/1 is ON.
2. If switch 2 is ON, light O:0/2 is ON.

This relationship among circuit elements is also equivalent to the OR in Boolean Algebra. The OR states that there is more than one path the electricity can follow. It is possible for the electricity to follow both paths at the same time.

Figure 6-6 shows how the elements of the program in Figure 6-5 would be hard wired to the PLC. The lights are clearly in parallel on the output side of the PLC.

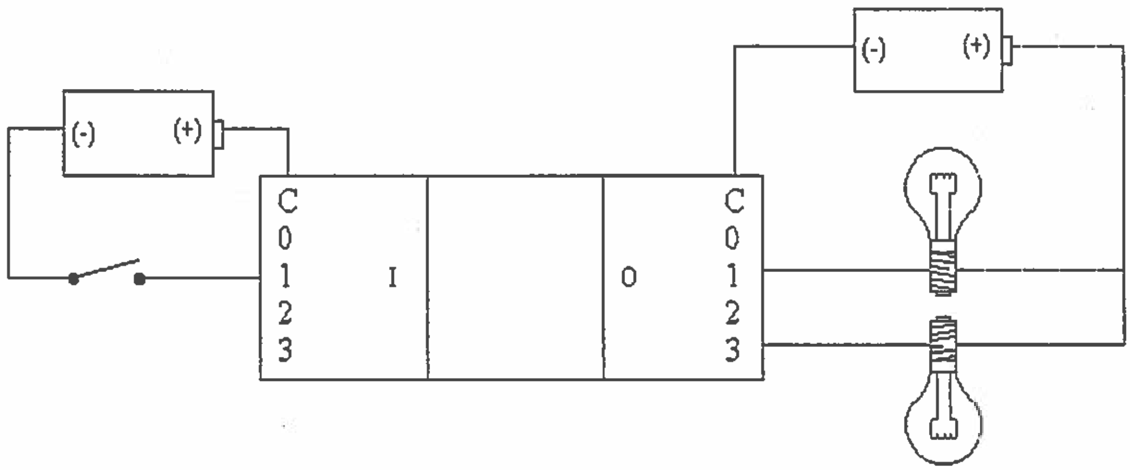


Figure 6-6
PLC Wiring for Figure 6-5

It is possible to convert any wiring diagram into a ladder logic diagram. First, it must be determined what combination of inputs turns on an output. Then each combination is written as rung of the program. Truth tables like those studied in Unit 6 can be used to sort through the various combinations when they become complex.

EXPERIMENT

Purpose

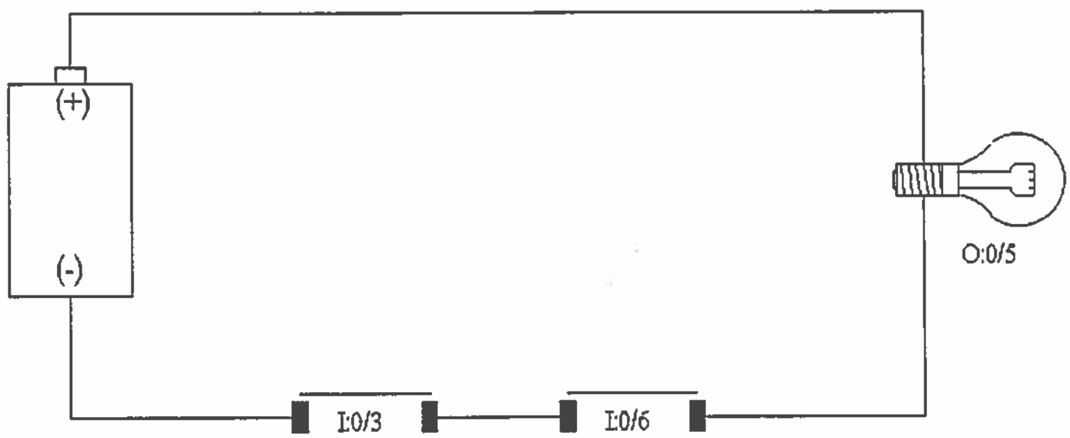
To convert circuit diagrams to ladder logic diagrams and to convert ladder logic diagrams to circuit diagrams.

Procedure

Refer to background materials to help you solve the problems that follow.

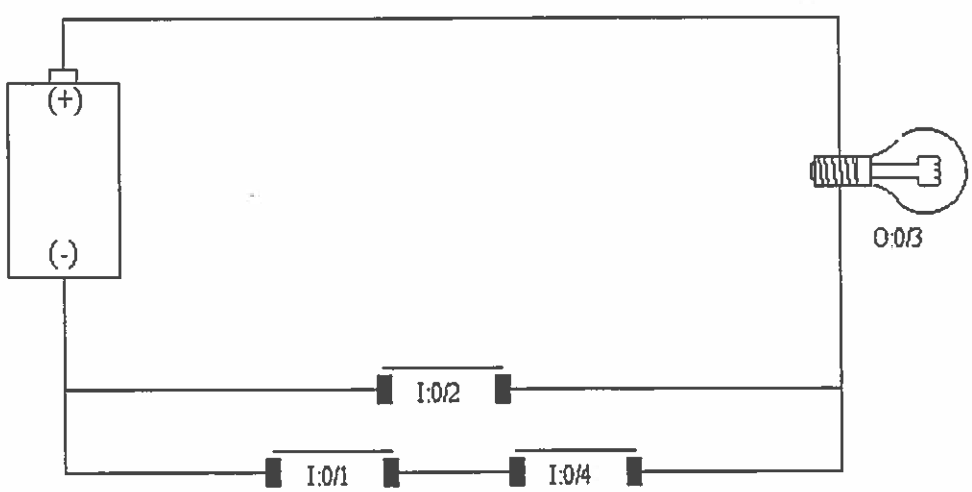
1. Redraw the circuit diagrams below as ladder logic diagrams.

a.



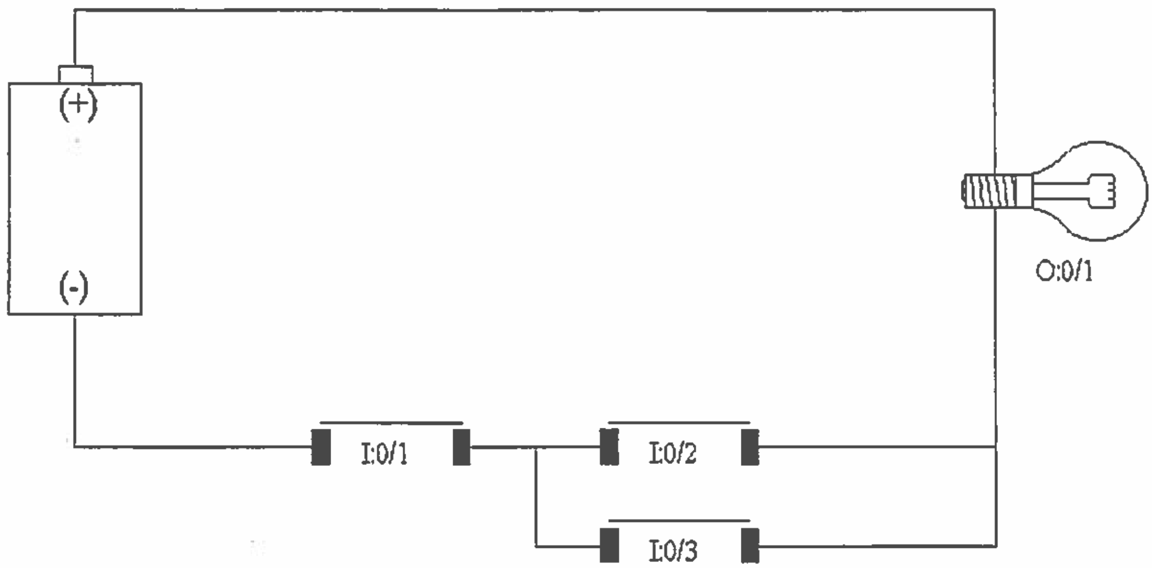
REFER TO ANSWER KEY

b.



REFER TO ANSWER KEY

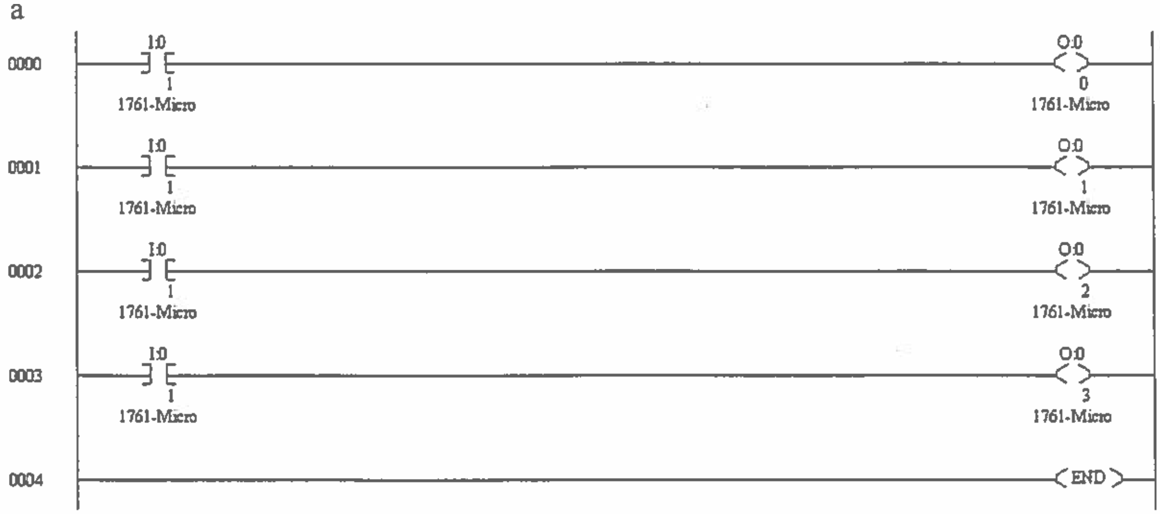
c.



REFER TO ANSWER KEY

2. Convert the ladder logic diagrams into circuit diagrams.

a



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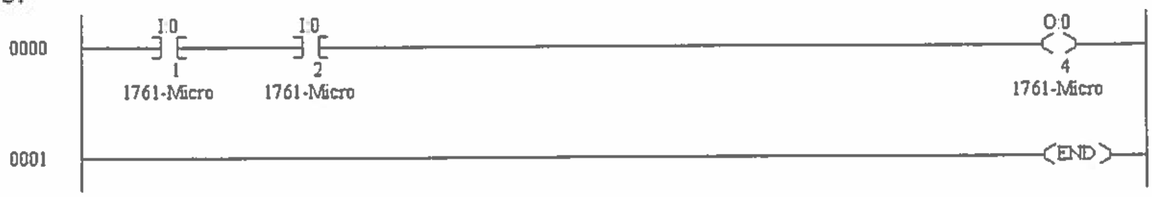
b.



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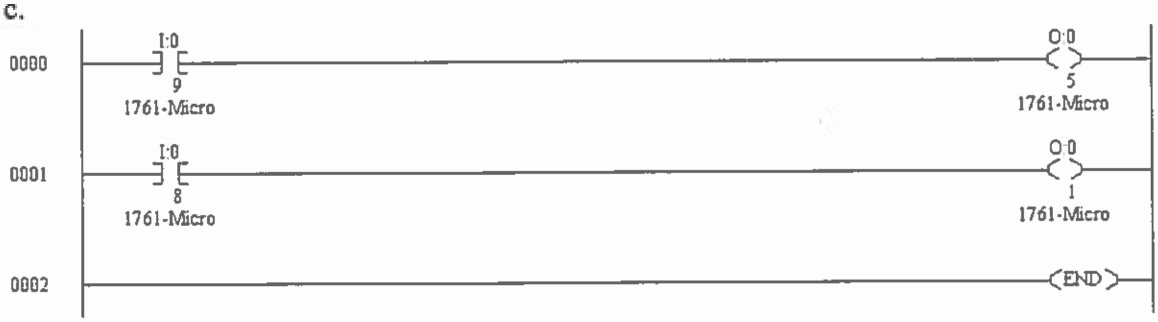
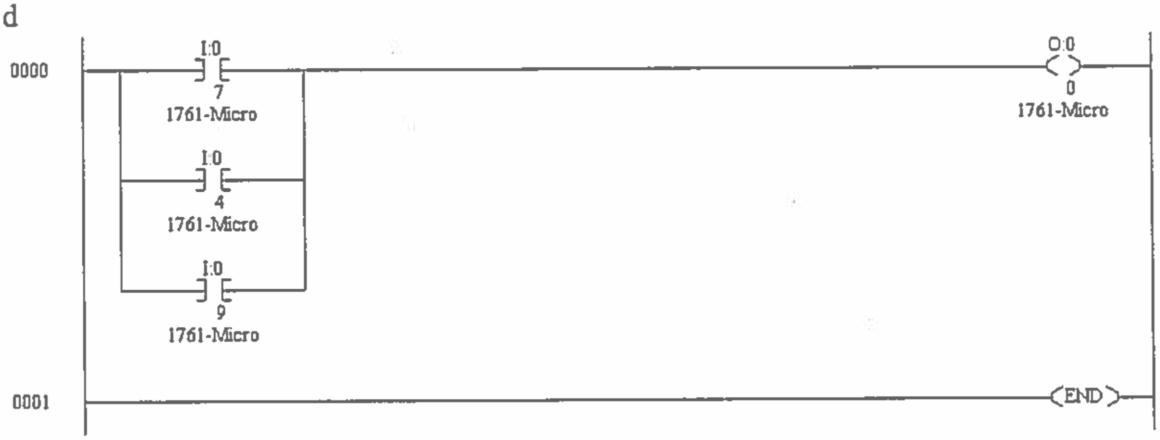
c.



REFER TO ANSWER KEY

d



REFER TO ANSWER KEY

3. Label the rungs in the program segment in Fig 6-7.

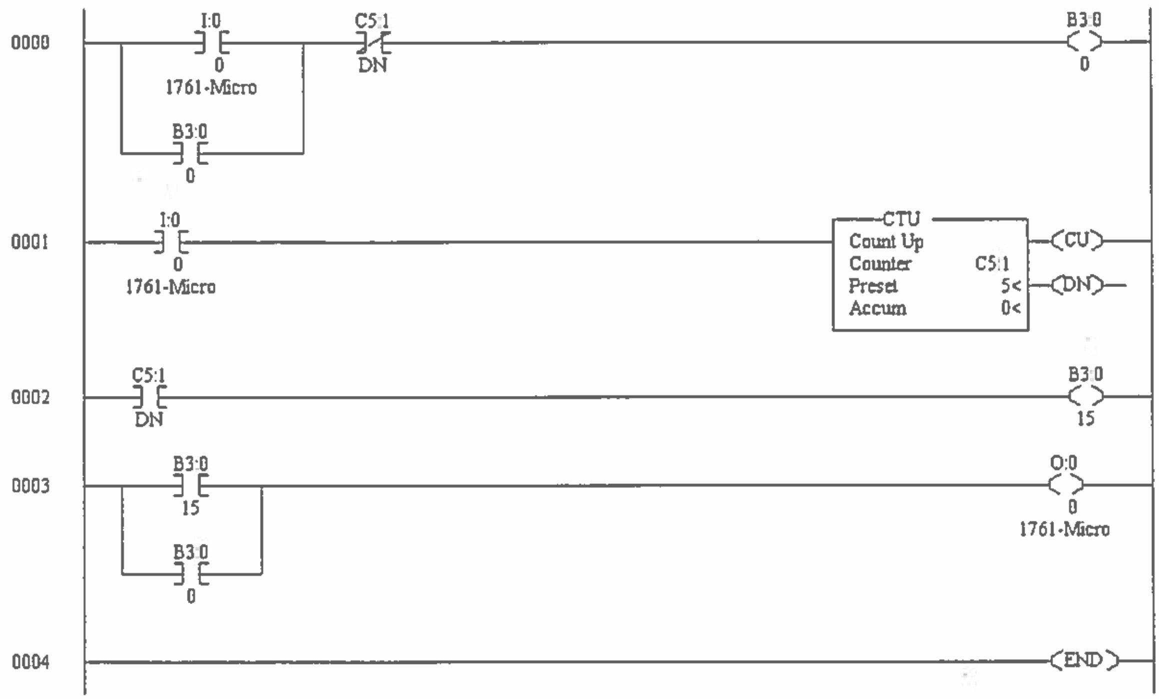


Figure 6-7

REFER TO ANSWER KEY

4. Now go to the end of this unit to check your answers. If you have any errors, review the background information and try the problem again.

5. Once you understand the answers to the three problems above, please complete the questions section below.

Questions

1. What is the difference between a ladder logic diagram and a circuit diagram?

   The circuit diagram shows how wires connect the elements of the circuit. The ladder logic diagram shows all possible conditions which exist in an electrical circuit.

2. Identify all the parts of the ladder logic diagram in Figure 6-8.

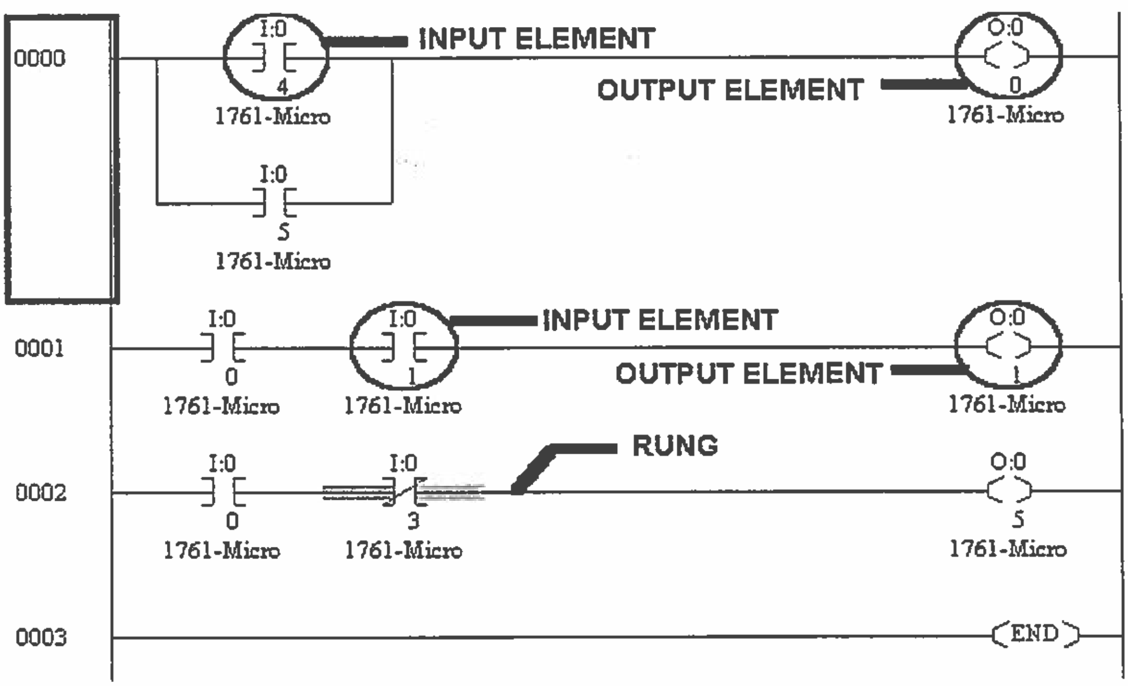


Figure 6-8.

3. Identify the following as TRUE or FALSE:

a. *False* There may be multiple outputs on a single ladder rung.

b. *True* There may be multiple inputs on a single ladder rung.

c. *True* A ladder logic diagram must have as many distinct elements as the circuit describes.

d. *False* There is no relationship between a ladder logic diagram and a Boolean equation.

e. *True* An output element may control as many rungs in a program as the programmer desires.

4. What is a rung in a ladder logic diagram?

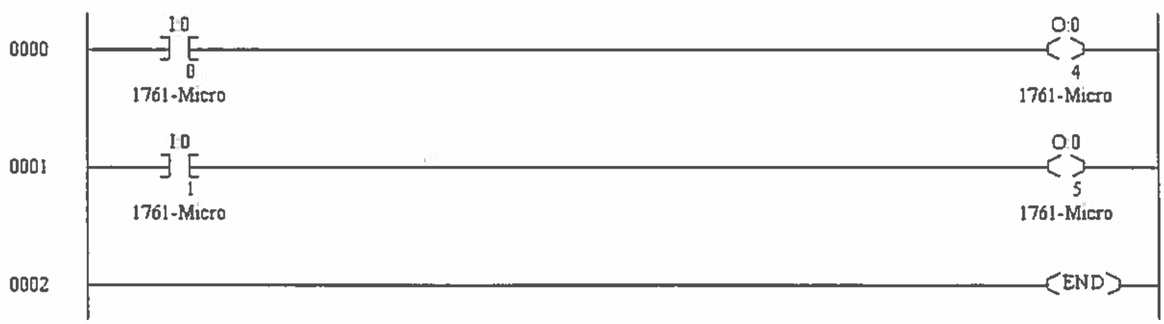
The rung is the horizontal line which carries the information about the inputs and outputs in the circuit.

5. What is the branch used to indicate in a program?

A branch is a parallel input in a circuit. It is the same as a Boolean OR relationship.

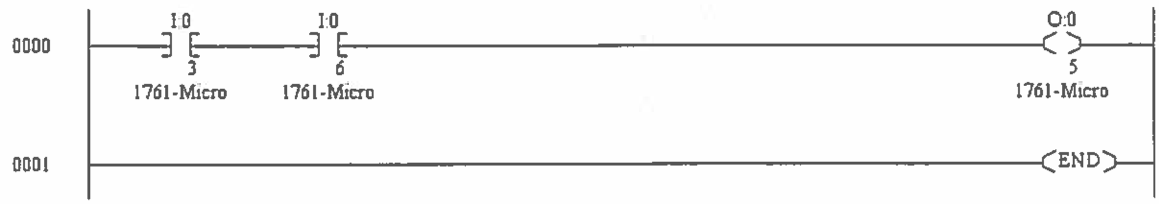
6. Draw a ladder logic diagram that will meet the following conditions:

When input I:0/0 turns on, output O:0/4 turns on.
When input I:0/1 turns on, output O:0/5 turns on.

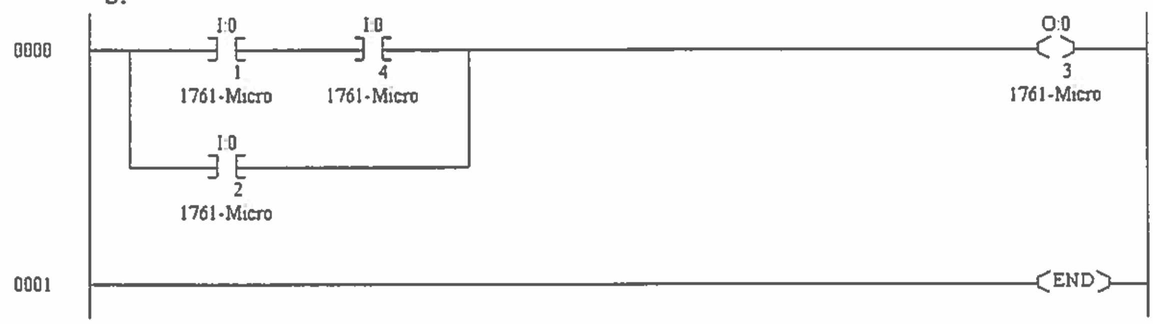


ANSWER KEY

a.

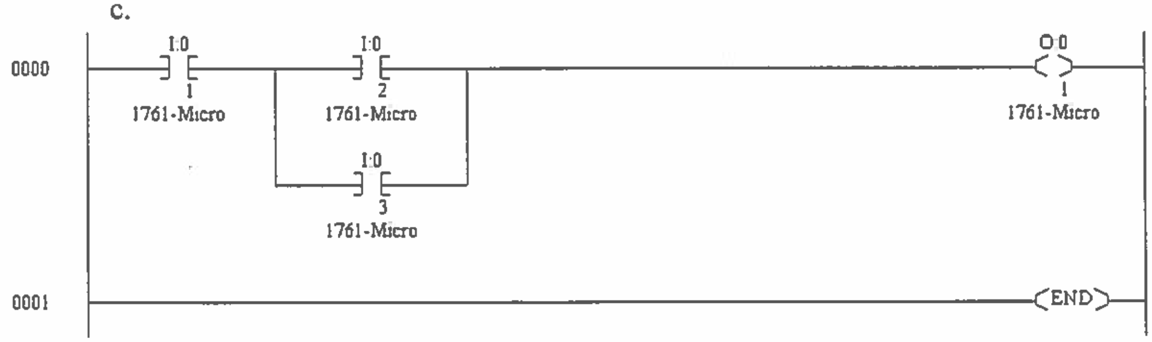


b.

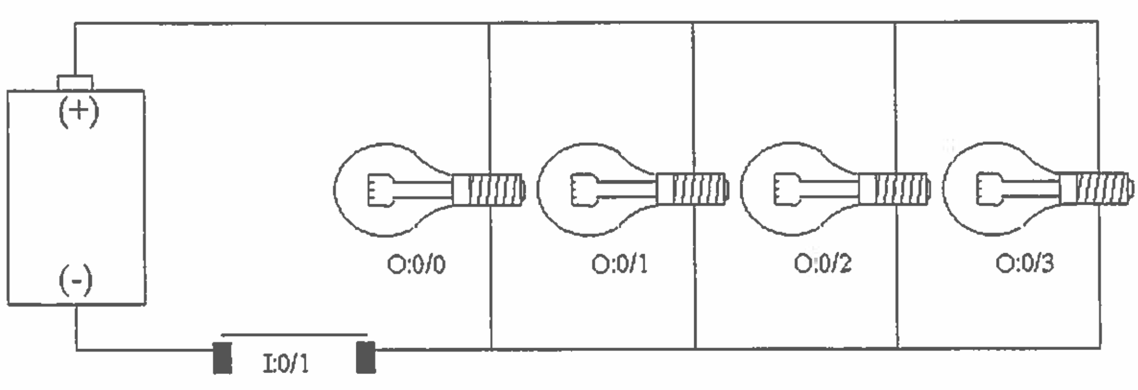


Note that this diagram has two inputs on the main rung and the single input on the branch. It is standard practice to place the simpler input statement on the branch and you should do so whenever you design a program.

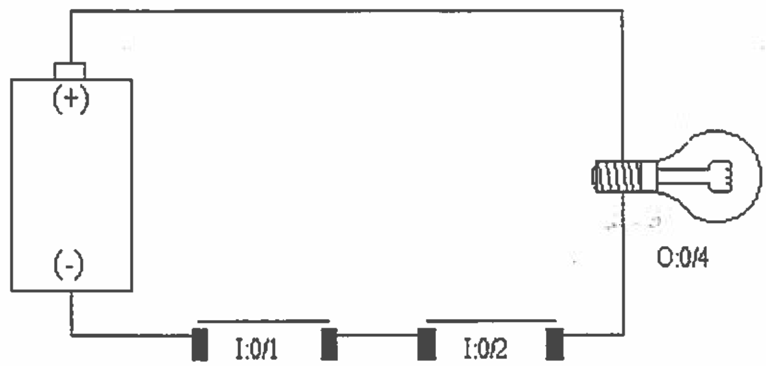
c.



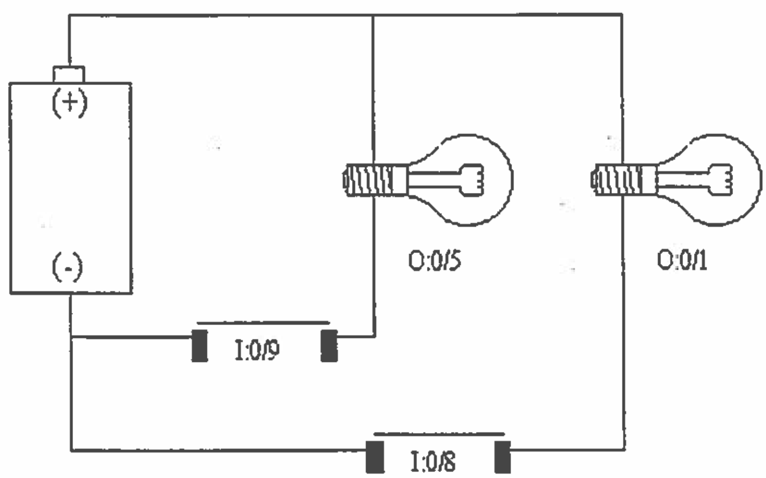
2. a.



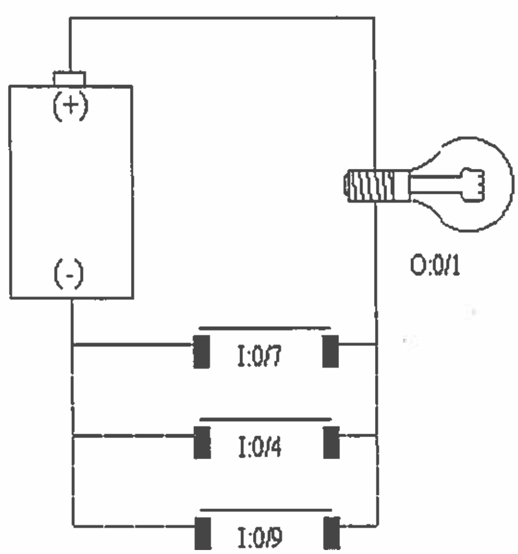
b.



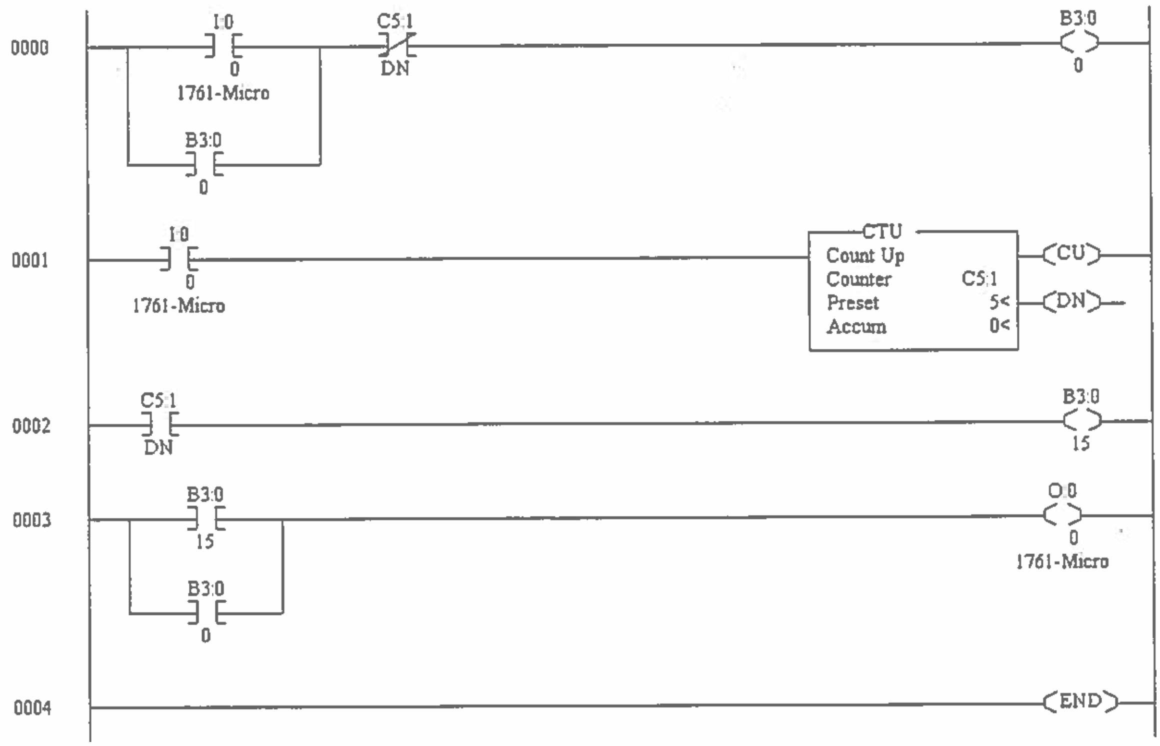
c.



d.



3.



0000
I:0
0
1761-Micro
C5:1
DN
B3:0
0
B3:0
0
0001
I:0
0
1761-Micro
CTU
Count Up
Counter C5:1
Preset 5<
Accum 0<
CU
DN
0002
C5:1
DN
B3:0
15
0003
B3:0
15
B3:0
0
O:0
0
1761-Micro
0004
END