SOLN

Direct Variation

Defn: When a change in one quantity causes a proportional change in another quantity, the quantities are directly proportional to one another. The two variables in this situation are said to be in direct variation.

Call your two variables x and y. The equation y/x = 6 states that "y varies directly as x" since the ratio of y to x never changes.

Solve $\frac{y}{x} = 6$ for the variable y: $x \cdot \frac{y}{x} = 6 \cdot x$ $\quad y = 6x$

↳ constant of variation

General equation of direct variation:

$$y = k \cdot x$$

where k = constant of variation $(k \neq 0)$

Ex 1: If y varies directly as x, and y = 8 when x = 12, find k and write an equation that expresses this variation.

Plan of Attack:

1. Plug the given values into the equation y = kx.
2. Solve for k.
3. Replace k with its value in the equation y = kx (This is your "equation of variation")

$y = k \cdot x$

$8 = k \cdot 12$

$k = \frac{8}{12} = \frac{2}{3}$

$$y = \frac{2}{3} \cdot x$$

Check: $8 \stackrel{?}{=} \frac{2}{3}(12)$

$8 = 8$ ✓

Ex 2: If y varies directly as x, and y=24 when x=16, find y when x=12.

$y = k \cdot x$

$24 = k \cdot 16$

$k = \frac{24}{16} = \frac{6}{4} = \frac{3}{2}$

Equation of variation: $y = \frac{3}{2}x$

When x=12, y= $y = \frac{3}{2}(12)$

$y = \frac{36}{2} = 18$

Ex 3: Hooke's Law F=kx

A principle of physics stating that the force (*F*) needed to either stretch or compress a spring by some distance (*x*) is directly proportional to that distance. Let *k* be the spring constant that relates the force being applied to the distance.

In other words, the weight hung on a spring (or force applied) varies directly to the distance a spring stretches. (Hang a heavier weight, spring will stretch further)

a. Suppose when a 7 kg weight is hung on a spring, the spring stretches to 50 cm. What is the distance when the weight is 9 kg?

→ eqn of variation

$F = k \cdot x$

$7 \text{ kg} = k \cdot 50 \text{ cm}$

$k = \frac{7}{50}$

$F = \frac{7}{50} \cdot x$

$\frac{9}{7/50} = \frac{\frac{7}{50} \cdot x}{7/50}$

$x = \frac{50}{7} \cdot 9 = 64.286 \text{ cm}$

b. If the distance is 8 inches when the weight hung on the spring is 3.5 kg, what is the weight when is the distance is 11 inches?

$F = k \cdot x$

$3.5 = k \cdot 8$

$k = \frac{3.5}{8}$

$F = \frac{3.5}{8} \cdot x$

$F = \frac{3.5}{8} \cdot 11 = 4.8 \text{ kg}$

c. How are direct variation and direct proportions related? Show how to solve Ex 3a using a proportion.

They are the same!

$\frac{7 \text{ kg}}{50 \text{ cm}} = \frac{9 \text{ kg}}{x}$

$7x = 50 \cdot 9$

$x = \frac{50 \cdot 9}{7} = 64.286 \text{ cm}$

* A common application of direct variation → [illegible]