College of Southern Idaho Summer Math Transition Course

Summer 2014

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Course Description:

This course will be setup as a competency based open lab which will be open entry and open exit. It will run from June 9 through July 31. It covers topics normally discussed in Math 015 and Math 025. For motivated students, this course can serve as a fast track through those courses. Successful participants will be prepared for Math 108 or Math 123 upon completion of the course.

Pre-requisites: Placement test score. Final course placement will be determined by the instructor.

Required Textbooks and Supplies:

- No Textbook is required.
- Calculator: A scientific calculator is highly recommended. The TI-30X is appropriate. Graphing calculators, such as TI-82 and higher, are not allowed on tests. Likewise, calculators on smartphones are not allowed on tests.

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Course Objectives:

- Whole number operations, exponents, order of operations agreement, prime factoring, applications, and solving equations of one variable.
- Introduction to integers.
- Fractional operations with applications, combining like terms, solving equations of one variable, exponents, complex fractions and order of operations.
- Decimals and Real Numbers with radicals.
- Variable Expressions.
- First degree Equations.
- Ratios, Rates, Unit Rates, and Proportions with applications.
- Percentages with applications.
- Factoring.
- Rational Expressions.
- Simple Graphing properties
- Introduction to radicals.

Policies and Procedures:

Attendance: This course is set up as an open lab. Students may come as often as needed. The lab will be open 1:00 – 5:00 pm, Monday through Thursday, and facilitated by math faculty.

You must have a COMPASS placement test score upon entering the course.

Disability Statement:

Any student with a documented disability may be eligible for reasonable accommodations. To determine eligibility and secure services, students should contact Student Disability Services at their first opportunity after registration for a class(es). Student Disability Services is located on the second floor of the Taylor Building on the Twin Falls Campus. 208.732.6260 or e-mail Marita DeBoard at mdeboard@csi.edu.

Outcomes Assessment:

After completing all units, or after the course ends, the student will retake the Compass for placement.

Course Setup:

Units: There will be six units that students can complete. They are: (1) Real Numbers; (2) Variable Expressions; (3) Equations and Inequalities; (4) Factoring; (5) Rational Expressions; and (6) Graphing. Each unit will consist of WebAssign assignments, videos, handouts, and worksheets.

Tests: Before each unit, students will take a pre-test on the material covered in that unit. If the student receives 80% or better on that pre-test, they may skip that unit and proceed to the next one. Otherwise, the student will complete the unit and retake test.

Math 015/025 Assessments: After students have successfully completed or tested out of all the units or when the course ends, whichever comes first, they will retake the Compass exam for placement.

Help Opportunities

Students are strongly encouraged to ask questions during class and office hours. The Learning Assistance Center, http://www.csi.edu/ip/adc/lap/index.htm, provides drop-in tutoring and other aids for students.

CSI E-mail

Since email is the primary source of written communication with students, all registered CSI students get a college email account. Student e-mail addresses have the following format: <address>@eaglemail.csi.edu where <address> is a name selected by the student as a part of activating his/her account. Students activate their accounts and check their CSI e-mail online at http://eaglemail.csi.edu. Instructors and various offices send messages to these student accounts. Students must check their CSI e-mail accounts regularly to avoid missing important messages and deadlines. At the beginning of each semester free training sessions are offered to students who need help in using their accounts.

CSI CAMPUS SECURITY

The College of Southern Idaho is committed to providing safe campuses for all students. Currently in place is an Emergency Notification System (RAVE) that provides information relating to an emergency on any CSI campus. This information is delivered electronically and can be received by all phone numbers and internet-equipped computers identified by the student. Registration is automatic when students register and contact information can be customized online (http://www.csi.edu/alert/) as necessary. The Twin Falls campus is also equipped with an Emergency Warning “Siren” that can be heard outside of buildings across campus. In the event of a signal, students arriving on campus should leave, and others should proceed with caution to avoid the emergency area. Students are encouraged to report any emergency (medical, criminal, behavioral, etc.) that is cause for action. Do this by calling 911 regardless of which campus you are on. If you are on the Twin Falls campus also calling CSI Campus Security at 732-6605 after placing the 911 call (the Twin Falls campus has security personnel available 24/7).

COLLEGE OF SOUTHERN IDAHO MATH TRANSITION - UNIT 1 REAL NUMBERS

I. Whole Numbers

1. Answer the following:
   a. What is a natural number? List the first 10 natural numbers.
   b. What is a whole number? List the first 10 whole numbers.

   c. What are the first twelve place values?

   d. What is the standard form of a number? What is the expanded form of a number?

   e. What are the rules for rounding?

   f. List the different types of graphs described on pages 8 and 9, and give a brief description of each.

2. Use your new knowledge to respond to the following prompts:

   a. Graph the number on the number line:

      i. 3 ii. 4

      iii. 0 iv. 7

   b. On the number line, which number is:

      i. 4 units to the left of 12?

      ii. 8 units to the right of 23

   c. Place the correct symbol, < or >, between the two numbers:

      i. 53 42

      ii. 4,080 4,800

      iii. 12,134 12,143

   d. Write the number in words:

      i. 332

      ii. 65,123

      iii. 7,024,905

   e. Write the number in standard form:

i. two hundred seventy-three

ii. three hundred forty thousand five hundred eleven

iii. two million thirty thousand twelve

f. Write the number in expanded form:

i. 5,432

ii. 35,247

iii. 154,306

g. Round the number to the given place value:

i. 4,122; tens

ii. 419,678; thousands

iii. 84,123; ten thousands

3. What is addition?

4. Add:

a.
$$\begin{array}{r} 8 \\ +\ \ 5 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 121 \\ +\ \ 322 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 4,037 \\ 3,342 \\ +\ \ 5,169 \\ \hline \end{array}$$

d. 456 + 1,291 + 10,000 =

5. There are several phrases which indicate addition:

a. added to	4 added to 9	4 + 9
b. more than	4 more than 9	9 + 4
c. the sum of	the sum of 4 and 9	4 + 9
d. increased by	4 increased by 9	4 + 9
e. the total of	the total of 4 and 9	4 + 9
f. plus	4 plus 9	4 + 9

6. a. What is 801 increased by 212?

b. What is 654 added to 7,293?

7. Properties of Addition:
Addition Property of Zero: the sum of any number and zero is the number
Examples: 8 + 0 = 8, 0 + 9 = 9, 0 + 12 = 12, 0 + 0 = 0, etc.
Commutative Property of Addition: Two numbers can be added in either order.
Example: 3 + 4 = 7 & 4 + 3 = 7, so 3 + 4 = 4 + 3

Associative Property of Addition: When adding 3 or more numbers, we can group in any order.
Example: $(2+3)+4=5+4=9$, $2+(3+4)=2+7=9$, so
$$(2+3)+4=2+(3+4)$$

8. Identify the property that justifies the statement:
   a. $11+(13+5)=(11+13)+5$

   b. $189+256=256+189$

   c. $182+0=182$

9. What is an equation?

10. What is a solution of an equation?

11. a. Is 5 a solution to the equation $x+4=9$?

    b. Is 8 a solution to the equation $m+9=19$?

    c. Is 13 a solution to the equation $y-7=6$?

12. What is subtraction?

13. Subtract:

    a. $12-7=$ b. $762-659=$

14. There are several phrases which indicate subtraction:
    a. minus 10 minus 3 $10-3$
    b. less 10 less 3 $10-3$

c. less than	3 less than 10	$10 - 3$
d. the difference between	the difference between 10 and 3	$10 - 3$
e. decreased by	10 decreased by 3	$10 - 3$
f. subtract … from	subtract 3 from 10	$10 - 3$

15. Find the difference between 168 and 32.

16. What is 48 less than 96?

**Note: Subtraction doesn't have the same properties as addition.
Example: $5 - 3$ is not the same as $3 - 5$

17. Find the perimeter of a rectangle that has a length of 22 ft and a width of 18 ft.

18. What is multiplication?

19. Multiply.

a. $9 \cdot 32$ b. $4(362)$ c. $(12)(15)$

20. Find the product of 800 and 3.

**Note that 4*x* means "4 times *x*."

21. Is 5 a solution to the equation $4x = 20$?

22. Is 23 a solution to the equation $3z = 96$?

23. What is an exponent? What does 3^4 mean?

24. Write the following in exponential form.

a. $2 \cdot 2 \cdot 2 \cdot 2$ b. $3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

c. $x \cdot x \cdot x \cdot x$

25. Evaluate.

a. 9^2 b. 3^3 c. 2^5 d. 10^6

26. What is division?

27. Divide.

a. $18 \div 6$ b. $248 \div 4$ c. $\frac{80}{16}$

28. Is 48 a solution of the equation $\frac{x}{12} = 4$?

29. Review natural numbers.

30. A **factor** of a natural number is divides the number evenly.

Example: 1, 2, 3, and 6 are factors of 6 since they divide 6 evenly.

31. Find all the factors of 12.

$12 \div 1 = 12$

$12 \div 2 = 6$

$12 \div 3 = 4$

$12 \div 4 = 3$ …

We see that the factors of 12 are 1, 2, 3, 4, 6, and 12.

32. A **prime** number is a natural number greater than 1 whose only factors are 1 and the number itself.

Examples: 2, 3, 5, 7, 11, 13, 17, 19, 23,…

33. Every number bigger than 1 has prime factors. For example, 2 and 3 are factors of 12.

34. The Fundamental Theorem of Arithmetic states that **every** number can be written as a unique product of prime factors (up to ordering of the factors).

Example: $12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$

35. The expression of a number as a product of prime factors is known as the **prime factorization** of the number.

36. Find the prime factorization of the following

a. 16 b. 24 c. 45 d. 70

37. The Order of Operations Agreement:

"**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally"

1. **P =** parenthesis. Perform the operations in grouping symbols (such as ( ), [ ], { }, **| |**, and the fraction bar) first.

2. **E** = exponents. Simplify exponential expressions.

3. **M & D** = multiplication and division. Do multiplication and division as they occur from left to right.

4. **A & S** = add and subtract. Do addition and subtraction as they occur from left to right.

__NOTE: ORDER MATTERS__

38. Evaluate:

    a. $100 \div (50 \times 2) =$_____

    b. $(100 \div 50) \times 2 =$_____

    c. $100 \div 50 \times 2 =$_____

    d. $24 - 18 \div 3 + 2 =$_____

    e. $3^2 \cdot 2^2 + 3 \cdot 2 =$_____

    f. $18 + 3(7) =$_____

    g. $2(5 - 3) + 6 =$_____

39. Evaluate the expression for the given values of the variables:

    a. $x + 5y$; for $x = 7$ and $y = 4$

    b. $m - n^2$; for $m = 20$ and $n = 3$

    c. $(x + y)^2 - 2xy$; for $x = 3$ and $y = 2$

II. Integers

1. We use numbers to quantify aspects of our lives every day. For example, we use numbers to tell us how hot or cold something is, e.g. $100°$ C is the temperature at which water boils, and $0°$ C is the temperature at which water freezes.

2. How do we indicate temperatures that are colder than $0°$C?
   What does "5 below zero" mean?

3. Numbers that are smaller than zero are called **negative numbers**. Numbers greater than zero are positive numbers.

   a. Negative numbers are indicated by the negative sign $(-)$.

4. Review : The Number Line

1. The whole numbers, 0, 1, 2, 3,…, together with their negative counterparts, $-1, -2, -3, \ldots$, make up the integers.

6. a. On the number line, what number is 6 units to the left of 4?

   b. What number is 5 units to the left of 2?

7. Recall that numbers get larger from left to right on the number line. Place the correct symbol, $<$ or $>$, between the two numbers:

a. $3 \quad 1$	b. $3 \quad 8$	c. $-3 \quad 1$
d. $3 \quad -8$	e. $-3 \quad -1$	d. $-3 \quad -8$

8. How far away is 3 from 0 on the number line?

   How far away is -3 from 0 on the number line?

   Note that 3 is to the right of 0, and -3 is to the left of 0 on the number line.

9. Two numbers that are the same distance from 0, but on opposite sides of 0 on the number line are called opposites.

10. Find the opposite of the number:

a. 3	b. 15	c. 21
d. -5	e. -25	f. -128

11. The negative sign can be read "the opposite of."

   Example: $-(-3)$ is read as "the opposite of negative 3."

12. Evaluate the following:

   a. $-(-3)$

   b. $-(-16)$

   c. $-(-x)$

13. The distance of a number from zero is called the **absolute value** of the number. To indicate that we want to find the absolute value of a number we place the number between absolute value symbol, $|\ |$.

   Example: The distance from 3 to zero is 3, so $|3| = 3$.

14. Evaluate the following:

a. $|6|$ b. $|-6|$ c. $|-201|$

d. $-|6|$ e. $-|25|$ f. $-|-3|$

15. Place the correct symbol, $<, =$, or $>$, between the two numbers.

a. $|3| \quad |-4|$ b. $|-15| \quad |-8|$ c. $|-12| \quad |12|$

16. ***Adding Integers.*** Use a number line to add the following:

a. $3 + 2$

b. $-3 + 2$

c. $-3 + 5$

17. NOTE: The commutative property of addition says that $3 + 2 = 2 + 3 = 5$. Likewise, $-3 + 2 = 2 + (-3) = -1$. **Adding by a positive number is rightward movement, adding by a negative number is leftward movement.** Use a number line to add the following:

a. $2 + (-3)$

b. $(-3) + (-2)$

18. ***General Rule for Adding Two Integers.***

a. ***Same Sign:*** (Two positives or two negatives) Add the digits / Attach the sign

i. $9 + 16$

ii. $(-12) + (-22)$

iii. $(-15) + (-27)$

b. ***Different Signs:*** Subtract the digits / Attach the sign of the larger digit

i. $-30 + 16$

ii. $80 + (-58)$

iii. $(-12) + 12$

19. Add:
   a. $3 + (-5) + (-4)$

   b. $10 + (-14) + (-21) + 8$

   c. What is 4 increased by -23?

   d. Find the sum of $-3, -8$, and 12.

   e. Evaluate $-x + y$ for $x = -9$ and $y = -7$.

   f. Evaluate $-x + (-y) + z$ for $x = -2$, $y = 8$, and $z = -11$

20. ***The Addition Property of Zero.*** Add:
   a. $3 + 0$ b. $-3 + 0$ c. $x + 0$ d. $-x + 0$

21. ***The Commutative Property of Addition.*** Add:
   a. $(-8) + 15$ b. $15 + (-8)$

22. ***The Associative Property of Addition.*** Add:
   a. $(3 + 2) + (-12)$ b. $3 + \big(2 + (-12)\big)$

23. ***The Inverse Property of Addition.*** Add:
   a. $8 + (-8)$ b. $147 + (-147)$ c. $-x + x$

24. NOTE: Because of the last property, the opposite of a number is also called the **additive inverse** of the number.

25. ***Subtracting Integers.*** Read the following:
   a. $7 - 6$ b. $-4 - 1$ c. $3 - (-9)$ d. $-5 - (-2)$

26. Evaluate the following:
   a. $12 - 9$

   b. $12 + (-9)$

27. ***Rule for Subtracting Integers.*** Add the opposite (of second integer)

28. Subtract:

a. $42 - 12$

b. $15 - 18$

c. $-5 - 3$

d. $5 - (-3)$

e. $-5 - (-3)$

29. Simplify: $-4 - (-7) + (-3)$

30. What is 5 less than -9

Evaluate $-x - (-y)$ for $x = -4$ and $y = -8$.

31. Find the next three numbers in the pattern:

10, 8, 6, 4, 2, …

32. Finish the pattern:

$$2(4) = 8$$
$$2(3) = 6$$
$$2(2) = 4$$
$$2(1) = 2$$

$$2(0) =$$
$$2(-1) =$$
$$2(-2) =$$
$$2(-3) =$$
$$2(-4) =$$

33. Find the next three numbers in the pattern:

$-6, -4, -2, 0, \ldots$

34. Finish the pattern:

$$(-2)(3) = -6$$
$$(-2)(2) = -4$$

$$(-2)(1) = -2$$
$$(-2)(0) = 0$$

$$(-2)(-1) =$$
$$(-2)(-2) =$$
$$(-2)(-3) =$$
$$(-2)(-4) =$$

35. Rules for Multiplying Two Integers:
    a. **To multiply two integers with the same sign** multiply their digits (absolute value). The product is positive.
    b. **To multiply two integers with different signs** multiply their digits (absolute value). The product is negative.

36. Multiply:
    a. $-5 \cdot 7$

    b. $(-4)(-3)$

    c. $(6)(7)$

    d. $(-13)(4)$

    e. $(8) \cdot (-30)$

37. The integers have the same multiplication properties that the whole numbers do.
    a. **The Multiplication Property of Zero**
       Multiply:
       1. $3 \cdot 0 =$

       2. $(-3) \cdot 0 =$

    b. **The Multiplication Property of One**
       Multiply:
       1. $(-3) \cdot 1 =$

       2. $1 \cdot (-3) =$

    c. **The Commutative Property of Multiplication**
       Multiply:

1. $(-3)(2) =$

2. $(2)(-3) =$

d. **The Associative Property of Multiplication**

Multiply:

1. $(-2) \cdot (3 \cdot 5) =$

2. $(-2 \cdot 3) \cdot 5 =$

38. Multiply:

a. $4 \cdot (-8) \cdot 3 =$

b. $(-4)(-4)(2) =$

c. $(-2)(-3)(-2) =$

d. $(-2)(-3)(-4)(-3) =$

39. Evaluate the expression for the given values of the variables:

a. xy, for $x = -5$ and $y = -6$

b. $(-x)(y)$, for $x = -3$ and $y = -6$

40. Is -8 a solution to the equation $7x = -56$?

41. Is 0 a solution to the equation $-3a = -3$?

42. Review:

$$\frac{8}{2} = 8 \div 2$$

43. Note that division is the “reverse” operation of multiplication.

E.g. $8 \div 2 = 4$ because $4(2) = 8$

E.g. $15 \div 5 = 3$ because $3(5) = 15$

E.g. $12 \div 2 = 6$ because $6(2) = 12$

Example: $25 \div 5 =$______, because__________

Example: $\frac{12}{-3} =$ _______, because __________

Example: $-18 \div 3 =$_______, because __________

Example: $\frac{-32}{-4} =$ _______, because __________

44. **Division Properties**

a. You can divide 0 into as many parts as you want, you will always end up with 0. So, 0 divided by any integer (other than 0) is always 0.
$$\frac{0}{a} = 0$$
(If $a \neq 0$)

b. You can't divide a number into zero parts. (What would that even mean?) The expression
$$\frac{a}{0}$$
is undefined. Another way to think of this concept is that there is no number you can multiply zero by to get a. You can't divide by zero.

c. If a is any real number:
$$\frac{a}{1} = a$$

d. If $a \neq 0$, $\frac{a}{a} = 1$.

45. Evaluate:

a. $100 \div (50 \times 2) =$_____

b. $(100 \div 50) \times 2 =$_____

c. $100 \div 50 \times 2 =$_____

$d. -100 \div 4 + 3 =$_____

e. $2(3-5)-2=$_____

f. $4-(-3)^2=$_____

g. $4-(-2)^2+(-3)=$_____

h. $3\cdot(6-2)\div 6=$_____

i. $6-2(1-5)=$_____

j. $4\cdot 2-3\cdot 7=$_____

k. $3\cdot 2^3+5\cdot(3+2)-17=$_____

l. $-12(6-8)+1^3\cdot 3^2\cdot 2-6(2)=$_____

m. $16-4\cdot 8+4^2-(-18)-(-9)=$_____

46. Evaluate the expression for the given values of the variables:

a. $x+5y$; for $x=7$ and $y=4$

b. m^2-n^2; for $m=2$ and $n=3$

c. $(x-y)^2-4x$; for $x=3$ and $y=2$

d. $\frac{x-y}{z-w}$; for $x=4, y=6, z=8$, and $w=10$

e. $3-x+2y$; for $x=6$ and $y=-3$

III. Fractions

1. Review Prime Factorization

2. Find the prime factorization of the following:
   a. 24 b. 32 c. 132

3. What is a multiple? List the multiples of 3. List the multiples of 4. Which multiples do they have in common?

4. The **Least Common Multiple** (LCM) of two or more numbers is the smallest number that is a multiple each number.
   a. Example: 12 is the least common multiple of 3 and 4 since it is the smallest number that is a multiple of both 3 and 4.

5. To find the LCM of some numbers is found by:
   a. Find the prime factorization of each number.

   b. Circle the highest power of each prime factor.

   c. The LCM is the product of each prime that appears in any of the prime factorizations raised to the highest power in any of the factorizations.

      i. Example: Find the LCM of 6 and 8.
         P.F. of 6 = $2 \cdot 3$
         P.F. of 8 = $2 \cdot 2 \cdot 2 = 2^3$
         LCM = $2^3 \cdot 3 = 8 \cdot 3 = 24$
         The least common multiple of 6 and 8 is 24.

6. Find the LCM.
   a. 4, 6

   b. 3, 9

   c. 9, 15

   d. 24, 36

   e. 6, 9, 15

7. The **Greatest Common Factor** (GCF) of two or more numbers is the biggest number that divides evenly into all the numbers.
    a. Example: 4 is the greatest common factor of 8 and 12 since 4 is the largest number that divides evenly into 8 and 12.

8. To find the GCF:
    a. Find the prime factorization of each number

    b. Circle the factors they have in common.

    c. The greatest common factor is the product of the prime factors that the numbers have in common.
    d. Example: Find the GCF of 16 and 24
    P.F of 16 = $2 \cdot 2 \cdot 2 \cdot 2$

    P.F of 24 = $2 \cdot 2 \cdot 2 \cdot 3$
    GCF = $2 \cdot 2 \cdot 2 = 8$

9. Find the GCF.
    a. 9, 15
    b. 6, 8
    c. 5, 7
    d. 32, 48
    e. 8, 12, and 24
    f. 8, 12, and 20

10. Parts of a fraction:

11. Examples:
    1. $\frac{1}{2}$

    2. $\frac{3}{4}$

    3. $\frac{2}{3}$

    4. $\frac{3}{5}$

12. ***Improper Fractions.*** What would $\frac{4}{3}$ be?

13. Examples:
    1. $\frac{4}{3}$

    2. $\frac{5}{2}$

14. ***Mixed Numbers.*** Note that $\frac{5}{2}$ pies is the same as two full pies and half another, or two and a half pies. The mixed number $2\frac{1}{2}$ is read two and a half.

    Be careful: $2\frac{1}{2}$ is **not 2 times $\frac{1}{2}$. $2\frac{1}{2}$ is actually $2 + \frac{1}{2}$.

    1. Write the improper fractions as a mixed number or a whole number.
        i. $\frac{5}{2}$
        ii. $\frac{31}{3}$
        iii. $\frac{15}{15}$

    2. Write the mixed number or whole number as an improper fraction.
        i. $2\frac{1}{3}$
        ii. $11\frac{2}{5}$
        iii. 9

15. ***Equivalent Fractions.*** Let's look at $\frac{1}{2}$ a pie, and $\frac{2}{4}$ a pie.

16. Note that if you divide the numerator and denominator of $\frac{2}{4}$ by 2 the result is $\frac{1}{2}$.

17. If you multiply or divide both the numerator and the denominator by the same number, the result is an equivalent fraction.

18. Write an equivalent fraction with the given denominator.
    1. $\frac{1}{2} = \frac{}{12}$

2. $\frac{3}{5} = \frac{}{15}$

3. $4 = \frac{}{5}$

19. To write a fraction in simplest form:

   1. Find the Greatest Common Factor (GCF) of the numerator and denominator.
   2. Divide the numerator and denominator by the GCF found in the first step.

20. Write the fraction in simplest form.

   1. $\frac{4}{12}$

   2. $\frac{6}{8}$

   3. $\frac{6}{21}$

   4. $\frac{0}{8}$

21. ***Order Relations between Fractions.*** Which is bigger, $\frac{1}{2}$ or $\frac{3}{4}$?

   1. Which is bigger, $\frac{2}{4}$ or $\frac{3}{4}$?
   2. To decide the order relation between fractions, first write the fractions with a common denominator. Whichever fraction has the larger numerator after this step is the larger fraction.
   3. To write the fractions with a common denominator, find the least common multiple (LCM) of the denominators. This will be the common denominator you can use, or the least common denominator (LCD).

22. Place the correct symbol, <, =, or >, between the two numbers.

   1. $\frac{3}{4} \qquad \frac{5}{6}$

2. $\frac{5}{7}$ $\frac{2}{3}$

3. $\frac{1}{10}$ $\frac{1}{100}$

23. ***Multiplying Fractions.***

a. What is half of 4?

b. What is half of 10?

c. What is half of one-half?

d. What is half of three-fourths?

e. Note: Taking one-half of three-fourths is the same as multiplying one-half by three-fourths.

f. To multiply two fractions together, multiply the numerators and the denominator, in other words "multiply across."

24. Multiply:

a. $\frac{1}{2} \cdot \frac{3}{4}$

b. $\frac{2}{3} \cdot \frac{5}{6}$

c. $-\frac{3}{4} \cdot \frac{5}{8}$

d. $\left(-\frac{5}{12}\right)\left(-\frac{4}{3}\right)$

e. $\frac{c}{9} \cdot \frac{d}{7}$

f. $2\frac{1}{4} \cdot 3\frac{1}{5}$ (Rewrite mixed numbers as improper fractions.)

g. $9 \cdot \frac{1}{3}$ (Rewrite integers as improper fractions.)

25. ***Division of fractions.***

a. Note that $9 \cdot \frac{1}{3} = 3$ and $9 \div 3 = 3$. So $9 \cdot \frac{1}{3} = 9 \div 3$.

b. $\frac{1}{3}$ is called the *reciprocal* of 3.

c. The reciprocal of a fraction is found by "switching" the numerator and denominator.

i. Find the reciprocal of the following:

1. $\frac{2}{3}$

2. $\frac{3}{4}$

3. $\frac{1}{2}$

4. 6

d. So, $9 \div 3 = 9 \cdot \frac{1}{3}$. Or 9 divided by 3 is the same as 9 times the reciprocal of 3.

e. Dividing by a number is the same as multiplying by the reciprocal of that number.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

f. Divide:

i. $\frac{2}{5} \div \frac{3}{4}$

ii. $\frac{1}{2} \div \frac{5}{6}$

iii. $\frac{5}{28} \div \left(-\frac{25}{42}\right)$

iv. $\frac{b}{6} \div \frac{d}{8}$

v. $-8 \div \frac{4}{5}$

vi. $3\frac{1}{3} \div 2\frac{2}{9}$

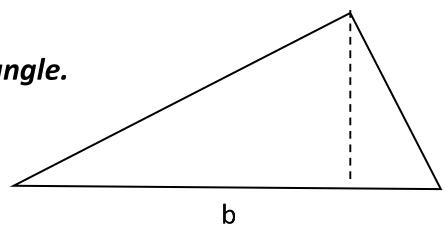
vii. $\frac{1}{5} \div 0$

26. Find xy and $x \div y$ if:
    a. $x = -\frac{1}{4}$ and $y = \frac{2}{7}$

    b. $x = 0$ and $y = \frac{1}{3}$

    c. $x = \frac{1}{3}$ and $y = 0$

27. ***Area of a Triangle.***



The formula for the area of a triangle is $A = \frac{1}{2}bh$

28. Find the area of a triangle with a base of 4 feet and a height of 15 feet.

29. A vegetable garden is in the shape of a triangle with a base of 21 feet and a height of 13 feet. Find the area of the vegetable garden.

30. ***Adding Fractions with the Same Denominator.*** To add fractions with the same denominator, add the numerators together. ****WARNING: DO <u>NOT</u> ADD THE DENOMINATORS****
    a. Example: $\frac{2}{5} + \frac{1}{5} = \frac{2+1}{5} = \frac{3}{5}$

    b. Add.
        i. $\frac{2}{7} + \frac{4}{7}$

        ii. $\frac{3}{8} + \frac{5}{8}$

        iii. $-\frac{3}{8} + \frac{5}{8}$

31. ***Adding Fractions with Different Denominators.*** To add fractions with different denominators, find the Least Common Denominator (LCD) and rewrite each fraction with the common denominator. Now that they have the same denominator, add numerators. ****WARNING: DO <u>NOT</u> ADD THE DENOMINATORS****

    a. Example: $\frac{1}{4}+\frac{1}{3}=\frac{3}{12}+\frac{4}{12}=\frac{7}{12}$

    b. Add.

        i. $\frac{4}{9}+\frac{1}{6}$

        ii. $\frac{7}{15}+\frac{9}{20}$

        iii. $-\frac{5}{12}+\frac{7}{8}$

        iv. $\frac{3}{8}+\frac{1}{2}+\frac{5}{12}$

        v. $1\frac{1}{10}+4\frac{3}{5}$

32. ***Subtraction of Fractions with the Same Denominator.*** To subtract fractions with the same denominator, subtract the numerators. ****WARNING: DO <u>NOT</u> SUBTRACT THE DENOMINATORS****

    a. Example: $\frac{5}{8}-\frac{3}{8}=\frac{5-3}{8}=\frac{2}{8}=\frac{1}{4}$

    b. Subtract.

        i. $\frac{5}{7}-\frac{2}{7}$

        ii. $\frac{4}{9}-\frac{5}{9}$

        iii. $-\frac{2}{3}-\frac{1}{3}$

        iv. $\frac{1}{8}-\left(-\frac{3}{8}\right)$

        v. $\frac{8}{d}-\frac{9}{d}$

33. ***Subtraction of Fractions with Different Denominators.*** To subtract fractions with different denominators, find the Least Common Denominator (LCD) and rewrite each fraction with the common denominator. Now that they have the same denominator, subtract numerators. ****WARNING: DO <u>NOT</u> SUBTRACT THE DENOMINATORS.***

   a. Example: $\frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{4-3}{12} = \frac{1}{12}$

   b. Subtract.

      i. $\frac{3}{4} - \frac{3}{8}$

      ii. $\frac{2}{5} - \frac{1}{2}$

      iii. $\frac{5}{8} - \left(-\frac{9}{10}\right)$

      iv. $4\frac{1}{2} - 1\frac{2}{5}$

      v. $8 - 4\frac{3}{7}$

34. A roofer and an apprentice are roofing a newly constructed house. In one day, the roofer completes $\frac{1}{3}$ of the job and the apprentice completes $\frac{1}{4}$ of the job.

   a. How much of the job did they complete?

   b. How much of the job remains to be done?

35. ***Exponents.***

   a. Review: $2^4 = 2 \cdot 2 \cdot 2 \cdot 2$

   b. Evaluate.

      i. $\left(\frac{3}{4}\right)^2$

      ii. $\left(\frac{2}{3}\right)^3$

      iii. $\left(-\frac{3}{5}\right)^3$

iv. $\left(-\frac{3}{5}\right)^3 \cdot \left(\frac{5}{6}\right)^2$

c. Evaluate x^2y^3 for $x = \frac{5}{8}$ and $y = \frac{4}{5}$

36. ***Complex Fractions.*** A complex fraction is a fraction that has a fraction in the numerator, the denominator, or both.

d. Examples:

$$\frac{\frac{1}{3}+\frac{1}{5}}{6}, \quad \frac{\frac{3}{4}}{\frac{7}{8}}, \quad \frac{\frac{3}{4}+\frac{5}{8}}{\frac{4}{9}}$$

e. Remember: the fraction bar means "divided by"

$$\frac{\frac{3}{4}}{\frac{7}{8}} = \frac{3}{4} \div \frac{7}{8}$$

f. Simplify.

i. $\frac{\frac{9}{16}}{\frac{3}{4}}$

ii. $\frac{2}{\frac{1}{3}+\frac{1}{4}}$

iii. $\frac{\frac{1}{2}+\frac{1}{4}}{\frac{2}{3}-\frac{3}{4}}$

Simplify.

g. $\frac{3}{5} \div \frac{6}{7} + \frac{4}{5}$

h. $-\frac{7}{18} + \frac{5}{6} \cdot \left(\frac{2}{3} - \frac{1}{6}\right)$

i. $\left(-\frac{2}{3}\right)^2 - \frac{7}{18} + \frac{5}{6}$

j. $\left(\frac{1}{3}\right)^2 \cdot \frac{14-5}{6-10} + \frac{3}{4}$

IV. Decimals

1. ***Decimal Notation.***
   a. Example: 312.468579

   b. What is the place value of the 4 in 7526.014?

2. ***Fractions as Decimals.***

$$\frac{7}{10} = 0.7, \qquad \frac{7}{100} = 0.07, \qquad \frac{7}{1000} = 0.007$$

   a. Write the fraction as a decimal:
      i. $\frac{4}{10}$

      ii. $\frac{37}{100}$

      iii. $\frac{101}{1000}$

      iv. $\frac{37}{1000}$

3. ***Decimals as Fractions.***
   a. Write the decimal as a fraction:
      i. 0.9

      ii. 0.21

      iii. 0.07

      iv. 0.0101

4. ***Order Relations.*** Note that 25 = 25.0 = 25.00 = 25.0000, etc. Likewise, it follows that 2.5 = 2.50 = 2.500 = 2.50000, etc. This is a fact we will use often when we compare two decimals.

   a. To compare decimals:
      i. Write the decimal part of each number so that each has the same number of decimal places.

ii. Compare the numbers.

b. Place the correct symbol, $<$ or $>$, between the two numbers:

i. 0.5 0.3

ii. 0.15 0.5

iii. 0.108 0.180

iv. 5.016 5.16

v. 6.8 60.8

c. Write the given numbers in order from smallest to largest:

i. 0.58, 0.508, 0.588

ii. 0.33, 0.344, 0.343, 0.304

iii. 0.06, 0.059, 0.061, 0.0061

5. ***Rounding.*** Decimals have the potential of continuing on forever. For example, $3.1415926535898\ldots$ Because of this, rounding, and understanding how others round, is very important.

a. To round to a given place value:

i. If the digit to the right of the given place value is less than 5, then that digit and all the digits to the right are dropped. In other words, "round down."

ii. If the digit to the right of the given place value is greater than 5, increase the digit in the given place value by one and drop all the digits to its right. In other words, "round up."

b. Round the number to the given place value.

i. 5.123; tenths

ii. 4.825; hundredths

iii. 3.1415926535898; hundredths

iv. 3.1415926535898; thousandths

v. 3.1415926535898; ten-thousandths

vi. 3.1415926535898; hundred-thousandths

vii. 3.1415926535898; whole number

6. ***Adding and Subtracting Decimals***. To add or subtract decimals, write the decimal part of each number so that each has the same number of decimal places. Follow the same rules as integers.

   a. Evaluate.

      i. $10.25 + 0.96$

      ii. $1.234 + 5.02$

      iii. $99 - 3.7$

      iv. $0.92 - 0.0037$

      v. $0.63 + (-0.52)$

      vi. $-1.24 - 3.6$

   b. If $x = 41.33$, $y = -26.095$, and $z = 70.08$, then find $x + y + z$.

   c. Is -1.3 a solution of the equation $x + 8.05 = 6.75$?

7. Suppose you make purchases for the following amounts: \$4.32, \$15.12, and \$20.75. How much money did you spend?

8. In Twin Falls, the average high temperature in August was $85.0°$F and the average high temperature in January was $34.9°$F. How many degrees did the average high temperature fall from August to January?

9. ***Multiplying Decimals.***

   To multiply decimals,

   a. Multiply the numbers as you would whole numbers.
   b. Count the number of decimal places in each factor of the product.
   c. Write the decimal point in the product so that the number of decimal in the product is the sum of the numbers of decimal places in the factors.

10. Multiply:

    d. $(2.8)(0.2)$

    e. $(8.29)(0.004)$

    f. $(-2.5)(3.2)$

    g. $(-1.3)(-2.01)$

11. ***Review.*** $\frac{3.0}{1.5} = \frac{3.0 \times 10}{1.5 \times 10} = \frac{30}{15} = 2$

12. ***Dividing Decimals.***

    To divide decimals,

    a. Move the decimal point in the divisor to the right so that the divisor is a whole number.
    b. Move the decimal point in the dividend the same number of places to the right.
    c. Place the decimal point in the quotient directly above the decimal point in the dividend.
    d. Divide as you would with whole numbers.

13. Divide.

    a. $5.4 \div 0.9$

b. $83.08 \div 6.2$

c. $7.02 \div (-3.6)$

d. $(-52.8) \div (-9.1)$

e.

14. ***Fractions and Decimals.***

Remember that the fraction bar can be read "divided by." So the fraction $\frac{3}{4}$ can be read "3 divided by 4," or $3 \div 4$.

15. Write the following fractions as decimals:

a. $\frac{3}{4}$

b. $\frac{1}{2}$

c. $\frac{1}{3}$

d. $\frac{5}{9}$

e. $\frac{5}{11}$

f. $\frac{1}{7}$

g. $\frac{5}{6}$

College of Southern Idaho Math Transition - Unit 2 Variable Expressions

When we talk about unknown quantities we use variables, which are symbols such as letters of the alphabet, as substitutes.

1. Example: The cost of gas for an unknown quantity of gas depends on the price of gas. If gas is $3.50 per gallon, then the cost is given by $c = 3.5x$, where c is the cost and x is the amount of gas in gallons.

2. An expression with one or more variables is called a **variable expression**.
   Examples:

   a. $x^2 + 6x - 3$

   b. $x^2 + 2xy + y^2$

   c. $m - 1$

3. The **terms** of a variable expression are the individual addends.
   a. Example: The terms of $x^2 + 6x - 3$ are x^2, $6x$, and -3.

   b. The **variable terms** of a variable expression are the terms containing a variable.
   c. Example: The variable terms of $x^2 + 6x - 3$ are x^2 and $6x$.
      i. **Note: -3 is called a **constant term**.

4. Name the variable terms of the expression:
   a. $2x^3 - 8x + 32$
   b. $5 - x^4$
   c. $5y^7 - y^8 + 2xy - 18$

5. To evaluate a variable expression means to find the numerical value of the expression given certain values for the variables.

6. Evaluate:
   a. $a + 2b$ when $a = 4$ and $b = -3$
   b. $x^2 - 4xy$ when $x = 5$ and $y = 2$
   c. $\frac{x^2-y^2}{x+y}$ when $x = 4$ and $y = 6$

d. $m(m-n)+2n-p$ when $m=2, n=3$, and $p=4$.

7. **Like terms** are terms that have the same variable part.
   a. Example: What are the like terms?
      i. $3x^2-2x+5-x+x^2$
      ii. $2x+3y-2xy$

8. We use the **distributive property** to combine like terms:
   a. If a, b, and c are real numbers, then $a(b+c)=ab+ac$

   b. Simplify the following:
      iii. $5x+9x$
      iv. $-3a+8a$
      v. $8x+3y$

9. Like terms have the same properties of addition that the real numbers have:
   a. The Associative Property of Addition: If a, b, and c are real numbers, then
   $$(a+b)+c=a+(b+c)$$
      i. Example: $5x+8x+12x$
   b. The Commutative Property of Addition: If a and b are real numbers, then
   $$a+b=b+a$$
      i. Example: $3x+(-12x)$

   c. The Addition Property of Zero: If a is a real number, then
   $$a+0=0+a=a$$
      i. Example: $3x+0$

   d. The Inverse Property of Addition: If a is a real number, then
   $$a+(-a)=0$$
      i. Example: $3x+(-3x)$

10. Simplify the following:
    a. $3x+(-5y)+4x+y$
    b. $x^2-7x+(-5x^2)+5x$
    c. $5x^3-4x^2+3x-2$

11. Like terms have the same properties of multiplication that the real numbers have:
   a. The Associative Property of Multiplication: If a, b, and c are real numbers, then
   $$(ab)c = a(bc)$$
      i. Example: $4(3x)$

   b. The Commutative Property of Multiplication: If a and b are real numbers, then
   $$ab = ba$$
      i. Example: $(9x) \cdot 8$

   c. The Multiplication Property of One: If a is a real number, then $a \cdot 1 = a$.
      i. Example: $(8x) \cdot 1$

   d. The Inverse Property of Multiplication: If a is a real number, then
   $$a \cdot \frac{1}{a} = 1$$
      i. Example: $y \cdot \frac{1}{y}$

   e. Simplify the following expressions:
      i. $-5(7x)$
      ii. $(-4y)(-9)$
      iii. $\frac{1}{5}(5x^2)$
      iv. $12x\left(\frac{1}{12}\right)$

   f. Simplify the following expressions:
      i. $2(4 - 3x)$
      ii. $3(4x^2 + 6)$
      iii. $4x - 2(3x + 8)$
      iv. $5n - 7(7 - 2n)$
      v. $2(x - 4) - 4(x + 2)$
      vi. $4[x - 2(x - 3)]$
      vii. $2x - 3[x - (4 - x)]$

12. Translate into a variable expression:
   a. The sum of 4 times *x* and 7
   b. The product of 5 and the difference between *w* and 3
   c. The difference between *x* and the square of *x*
   d. The sum of one-fourth of *y* and 15
   e. 6 divided by the total of *m* and

f. The difference between 3 times the cube of *n* and the square of *n*
g. Three-fifths of the sum of *x* and 12

13. The variable isn't usually given to us. In these cases, we need to assign a variable to the unknown quantity.
    a. Example: 12 minus a number

       The unnamed number is the unknown quantity

       Assign a variable: *x*

       ***Hint:* Replace the phrase "a number" with *x* before translating it into a math expression

       12 minus *x* Translation: $12 - x$

14. Translate into a variable expression
    a. A number divided by 12
    b. Five-sixths of a number
    c. Nine more than a number
    d. The quotient of twice a number and seven
    e. The difference between three-fourths of a number and eight
    f. The quotient of nine and the sum of 5 and a number
    g. The difference between thirty and the ratio of a number to ten.

15. Translate into a verbal expression, then simplify:
    a. Eight time the difference between a number and three
    b. The difference between a number and 1 more than the number
    c. The sum of two-thirds of a number and five-ninths of the number

The endgame of all this is to be able to translate "real world" problems into math problems.

16. The length of a basketball court is 6 feet less than twice the width. Express the length of the pool in terms of the width.

17. A banker divided $8000 between two accounts, one paying 6% annual interest and the second paying 4% annual interest. Express the amount invested in the 6% account in terms of the amount invested in the 4% account.

18. ***Monomials.*** A monomial is a number, a variable, or a product of a number and variables.

19. ***Polynomials.*** A polynomial is a sum of monomials. (Remember that subtraction can be thought of as adding the opposite.)

20. ***Binomials.*** A binomial is a polynomial which, when reduced, is the sum (or difference) of two monomials

21. ***Trinomials.*** A trinomial is a polynomial which, when reduced, is the sum of three monomials.

22. ***Degree of a polynomial (one variable).*** The degree of a polynomial with just one variable is the greatest exponent on that variable in any of the terms. The degree of a nonzero constant is zero. (The polynomial 0 has no degree.)
    a. Find the degree of the polynomial:
        i. $3x^2 + 2x - 1$
        ii. 3
        iii. $y^4 - y^2$
        iv. $1 + x + x^2 + x^3 + x^4 + x^5$

23. ***Descending order.*** If the exponents of the variable decrease from left to right, the polynomial is written in descending order.

    **Note: Polynomials are usually written in descending order. When you check your answers in the book, they will be written in descending order.

    a. Are the following polynomials written in descending order? If not, rewrite the polynomials in descending order.
        i. $x^2 - 3x + 2$
        ii. $1 + x$
        iii. $1 - x^4 + 2x^3 + x^5$
        iv.

24. ***Adding Polynomials.*** To add polynomials, combine like terms.

a. Add.
   i. $(3x^2 + 6x - 1) + (4x^2 - x + 5)$
   ii. $(4x^3 + 5x + 2) + (1 + 2x - 3x^2)$
   iii. $(y^2 + 3y^3 + 1) + (-4y^3 - 6y - 3)$

25. ***Subtracting Polynomials.*** To subtract polynomials, add the opposite of the second polynomial. To find the opposite of a number or polynomial, multiply it by -1.
   a. Example:

$$\begin{aligned}(3x^2 + x) - (x^2 - 2x) &= (3x^2 + x) + (-1)(x^2 - 2x) \\ &= (3x^2 + x) + (-x^2 + 2x) \\ &= 2x^2 + 3x\end{aligned}$$

   b. Subtract.
      i. $(-3y^2 + y) - (4y^2 + 6y)$
      ii. $(m^3 + 2m - 1) - (m^3 - m + 3)$
      iii. $(4 - x - 2x^2) - (-2 + 3x - x^3)$

26. ***Exponents.***
   a. Evaluate.
      i. 4^2
      ii. x^3
      iii. $(-x)^2$
      iv. $-x^2$

   b. Evaluate.
      i. $x^3 \cdot x^2$

27. ***Multiplying Exponential Expressions. (Add powers of the same variable)***

$$x^m \cdot x^n = x^{m+n}$$

   a. Evaluate.
      i. $x^4 \cdot x^8 \cdot x^3$
      ii. $(6x^2)(3x^5)$
      iii. $(6x^4y)(-4y^5)$
      iv. $(7p^6q^4)(-5p^2q^9)$

28. Evaluate: $(x^2)^3$

29. ***Simplifying the Power of an Exponential Expression. (Multiply Powers Outside Parenthesis by Powers inside Parenthesis)***

$$(x^m)^n = x^{m \cdot n}$$

a. Evaluate.
   i. $(x^8)^9$
   ii. $6x(x^4)^6$

30. Evaluate: $(x^2y^4)^3$

31. ***Simplifying the Power of a Product.***

$$(x^m y^n)^p = x^{mp} y^{np}$$

a. Evaluate.
   i. $(3x^2y^3)^3$
   ii. $(-3x^3)^2$
   iii. $(3a^2)(2b^3)^4$
   iv. $(4x^2y^3)(2xy^4)^3$
   v. $(3ab^3)(-2a^2b^4)^3$

32. ***Review: Distributive Property.***

$$a(b + c) = ab + ac$$
$$(a + b)c = ac + bc$$

a. Multiply.
   i. $3(x + 4)$
   ii. $2(5x^2 - 18)$
   iii. $4x(3x + 5)$
   iv. $-2x(x - 12)$
   v. $3x^2(x + 1)$
   vi. $2x^2(x^2 - 4x + 3)$

33. We can also use the distributive property when both polynomials have more than one term.

a. Example: $(x + 2)(x + 3)$

   If we use ( ) as a place holder for $(x + 2)$, we get

$$
\begin{aligned}
(\quad)(x+3) &= (\quad)x + (\quad)3 \\
&= (x+2)x + (x+2)3 \\
&= x^2 + 2x + 3x + 6
\end{aligned}
$$

Simplifying, we get $x^2 + 5x + 6$.

b. Multiply.
   i. $(3x+5)(x-2)$
   ii. $(2x-3)(x^2+4x+3)$
   iii. $(3a-2)(6a^3+4a^2-3a)$

34. ***FOIL.*** Note above that when the factors were both binomials (had two terms) the product had four terms before simplification. These four terms were: the product of the First terms; the product of the Outer terms; the product of the Inner terms; and the product of the Last terms.

Example: $(x+2)(x+3)$

Product of the First terms	$(\boldsymbol{x}+2)(\boldsymbol{x}+3)$	$x \cdot x = x^2$
Product of the Outer terms	$(\boldsymbol{x}+2)(x+\mathbf{3})$	$x \cdot 3 = 3x$
Product of the Inner terms	$(x+\mathbf{2})(\boldsymbol{x}+3)$	$2 \cdot x = 2x$
Product of the Last terms	$(x+\mathbf{2})(x+\mathbf{3})$	$2 \cdot 3 = 6$

If we add these terms together we get

$$x^2 + 3x + 2x + 6 = x^2 + 5x + 6$$

a. This method is called FOIL, which stands for First, Outer, Inner, and Last. FOIL is a way to help us remember to distribute.

b. Multiply.
   i. $(3x+5)(x-2)$
   ii. $(7x-3)(5x-6)$
   iii. $(6w+4)(5w-2)$

35. ***Special Products.***

a. ***Product of Sum and Difference of the Same Terms.***

$$
\begin{aligned}
(a+b)(a-b) &= a^2 - ab + ab - b^2 \\
&= a^2 - b^2
\end{aligned}
$$

   i. Multiply.

1. $(x+3)(x-3)$
2. $(3x-4)(3x+4)$

b. ***Square of a Binomial.***

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2+ab+ab+b^2\\ &= a^2+2ab+b^2\end{aligned}$$

**Note: $(a+b)^2$ does NOT equal a^2+b^2

i. Expand.
1. $(x+5)^2$
2. $(4m-3n)^2$
3.

36. The length of a rectangle is $(2x+3)$ feet. The width of the rectangle is $(x-3)$ feet. Find the area of the rectangle in terms of the variable x.

37. The base of a triangle is $(4x)$ meters and the height is $(2x+5)$ meters. Find the area of the triangle in terms of the variable x.

38. ***Dividing Exponential Expressions.***

$$\frac{x^m}{x^n} = x^{m-n}$$

39. Simplify.

a. $\frac{p^9}{p^4}$

b. $\frac{x^5y^6}{x^3y^2}$

c. $\frac{r^5}{t^2}$

40. Simplify: $\frac{x^3}{x^3}$

41. ***Zero as an Exponent.*** If $a \neq 0$, then $a^0 = 1$. The expression 0^0 is not defined.

42. Simplify:

a. $(ab)^0$

b. $(100x^2y^6)^0$

43. Evaluate: $\frac{x^2}{x^5}$

44. ***Negative Exponents.***

$$x^{-n} = \frac{1}{x^n} \text{ and } \frac{1}{x^{-n}} = x^n$$

45. Evaluate:

a. 2^{-4}

b. $4x^{-3}$

c. $\frac{5}{a^{-2}}$

46. Evaluate: $\left(\frac{x^5}{y^3}\right)^2$

47. ***Simplifying the Power of a Quotient.***

$$\left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}$$

Simplify.

a. $\left(\frac{x^3}{y^4}\right)^{-2}$

b. $\left(\frac{6m^2n^3}{8m^7n^2}\right)^{-3}$

48. ***Rules of Exponents.***

$x^m \cdot x^n = x^{m+n}$ $\quad$ $(x^m)^n = x^{mn}$ $\quad$ $(x^my^n)^p = x^{mp}y^{np}$

$\frac{x^m}{x^n} = x^{m-n}$ $\quad$ $\left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}$ $\quad$ $x^{-n} = \frac{1}{x^n}$

$$x^0 = 1, x \neq 0$$

49. Simplify:

a. $(-3x^4y^{-5})(-2x^{-3}y^{-1})$

b. $\frac{3x^{-1}y^4}{6^{-1}x^3y^{-2}}$

c. $\left(\frac{8a^{-3}b^{-1}c^2}{12a^3b^{-3}c^{-2}}\right)^{-2}$

d.

50. ***Scientific Notation.*** In physics, biology, astronomy, and other sciences we often deal with extremely large or extremely small numbers.

    Example: Light travels approximately 16,000,000,000 miles in 1 day.

    These numbers are difficult to read and write. A more convenient way is to express the number in scientific notation.

51. A number is written in scientific notation if it is in the form $a \times 10^n$ where a is a number between 1 and 10 and n is an integer.

52. For numbers bigger than 10:
    a. Move the decimal to the right of the first digit.
    b. Count how many places the decimal has been moved.
    c. The number found in (b) is the exponent n.

53. Write in Scientific notation.
    a. 300
    b. 250,000
    c. 1,500,000,000,000,000

54. For numbers less than 1:
    a. Move the decimal to the right of the first nonzero digit.
    b. Count how many places the decimal has been moved.
    c. The exponent n is the negative of the number found in part (b).

55. Write in Scientific Notation.
    a. 0.3
    b. 0.000516

56. Write the number in decimal notation:
    a. 3.8×10^4
    b. 3.94×10^{-4}

57. Light travels approximately 16,000,000,000 miles in 1 day. Write this number in scientific notation.

58. The length of an average animal cell is 0.00003 meters. Write this number in scientific notation.

60. ***Dividing by a Monomial.*** To divide a polynomial by a monomial, divide each term by the monomial.

a. Divide

i. $\frac{8x^3-4x^2+6x}{2x}$

ii. $\frac{3x^2y^2+6xy}{3xy}$

iii. $\frac{24a^2b^2+16ab-4b}{8ab}$

61. ***Division of Polynomials.***

b. Dividend $=$ (Quotient) $\cdot$ (Divisor) $+$ Remainder

c. $\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$

d. Divide:

i. $\frac{12}{5}$

ii. $\frac{692}{25}$

iii. $\frac{x^3+2x^2-5x-6}{x+1}$

iv. $\frac{x^2+5x+8}{x+3}$

v. $\frac{3x^3+x^4+5}{x^2+1}$

vi. $(6x^3 + x^2 - 18x + 10) \div (3x - 4)$

vii. $(x^4 - x^2 - 6) \div (x^2 + 2)$

62. ***Square Roots.*** A square root of a positive number x is a number whose square is x. That is, if $a^2 = x$, then $\sqrt{x} = a$.

   a. Examples:

      i. A square root of 16 is 4, since $4^2 = 16$.

      ii. Another square root of 16 is -4, since $(-4)^2 = 16$.

   b. Note: the square root symbol $\sqrt{}$ is used to indicate the positive square root.

63. Evaluate:

   a. $\sqrt{64}$

   b. $\sqrt{144}$

   c. $-\sqrt{144}$

   d. $\sqrt{9} \cdot \sqrt{4}$

   e. $\sqrt{9 \cdot 4}$

64. ***Square Roots of Whole Numbers.*** The square root of a number that is not a perfect square can only be approximated. These numbers are irrational numbers.

   a. Example: $\sqrt{2} \approx 1.4142$ (≈ means "is approximately")

65. ***Simplifying a Radical Expression.*** A radical expression is in simplest form when the number under the radical sign, the radicand, contains no factor, other than 1, that is a perfect square.

   a. Example: $\sqrt{20}$ is not in simplest form because 4 is a factor of 20 and 4 is a perfect square.

66. ***Product Property of Square Roots.*** $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$

   a. Example: $\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$

67. To simplify a radical:

   a. Find the largest perfect square that is a factor of the radicand.

   b. Rewrite the radicand as a product of the perfect square found and another number.

   c. Use the Product Property of Square Roots to simplify.

68. Simplify.

      i. $\sqrt{8}$

      ii. $\sqrt{27}$

      iii. $\sqrt{300}$

iv. $\sqrt{28}$
v. $7\sqrt{48}$

69. *Note:* The square root of a negative number is not a real number.
    a. Example: $\sqrt{-9}$ is not a real number.

70. ***Simplifying variable radical expressions.*** For simplification purposes, we will ***assume*** *that all variable expressions represent positive real numbers*.

    a. ***Review:*** Simplify $(x^3)^2$

    b. Simplify: $\sqrt{x^6}$

    c. If n is an even integer, then $\sqrt{x^n} = x^{n/2}$.

71. Simplify.
    a. $\sqrt{a^8}$

    b. $\sqrt{16y^4}$

    c. $\sqrt{9x^2y^6}$

    d. $\sqrt{25(x+1)^2}$

    e. $\sqrt{a^9}$

    f. $\sqrt{8y^5}$

    g. $\sqrt{27x^3y}$

    h. $\sqrt{x^2 + 10x + 25}$

College of Southern Idaho Math Transition - Unit 3 Equations and Inequalities

1. What is an equation? Example: $x = 3$

2. What is a solution of an equation?

3. Is -6 a solution of $4x + 3 = 2x - 9$?

4. Is $-\frac{2}{3}$ a solution to the equation $4 - 6x = 9x + 1$?

5. What does it mean to solve an equation?

6. Two equations are **equivalent** if they have the exact same solutions.

7. Addition / Subtraction Property of Equations:

8. Solve the following equations:
   a. $x + 3 = 5$
   b. $x - 4 = 6$
   c. $y + \frac{1}{2} = \frac{1}{4}$
   d. $x - \frac{1}{4} = \frac{5}{6}$
9. Multiplication / Division Property of Equations:
   a. $3x = 6$
   b. $\frac{1}{4}x = 6$
   c. $\frac{4}{5}x = 28$
   d. $4x - 8x = 16$
   e. $5y + 2y = 42$

We encounter percentages every day. Whether shopping or putting together an investment portfolio, they make up a good part of our lives. So, being able to translate problems involving percentages into math problems is a desirable skill.

$\boldsymbol{PB = A}$, P = percent as a fraction or decimal; B = base, or the number you are taking the percent of; A = Amount, or the result of taking the base.

E.g. 50% of 20 is 10 because $0.5(20) = 10$

Notes: (a) If solving for A or B, convert the percent to a decimal or fraction to put into the equation. (b) If solving for P, after the equation is solved convert the decimal or fraction to a percent.

10. What is 25% of 80?
11. 48 is 12% of what?
12. What is 20% of 40?
13. What percent of 125 is 50?

14. 18 is what percent of 9?
15. 3 is 20% of what?
16. 9 is 40% of what?

For some percentages, the fraction form of the percent is easier to work with:

17. $33\frac{1}{3}\% = \frac{1}{3}$, $66\frac{2}{3}\% = \frac{2}{3}$, $16\frac{2}{3}\% = \frac{1}{6}$, and $83\frac{1}{3}\% = \frac{5}{6}$
    What is $33\frac{1}{3}\%$ of 30?
18. A fire department received 24 false alarms out of a total of 200 alarms. What percent of the alarms were false alarms?

19. Each year 184 million pounds of lobster are caught in the United States and Canada. Twenty-five percent of this amount is sold live. How many pounds of lobster are sold live each year?

20. Use the Addition and Multiplication Properties of Equations to solve the following equations:
    a. $4y + 3 = 11$
    b. $3x + 1 = 10$
    c. $3v - 7 = -19$
    d. $-35 = -6b + 1$
    e. $9x - 4 = 0$
    f. $2y - 9 = 12$
    g. $2y + \frac{1}{3} = \frac{7}{3}$

22. ***Clearing Denominators.*** Using the Multiplication Property of Equations, when dealing with fractions in an equation we can "clear the denominators." The result is an equivalent equation without any fractions. To clear the denominators:
    a. Find the LCD of all the fractions in the problem.
    b. Use the Multiplication Property of Equations and multiply both sides by the LCD found in part (a).

23. Solve the following equations:
    a. $2y + \frac{1}{3} = \frac{7}{3}$
    b. $\frac{5}{4}x + \frac{2}{3} = \frac{1}{4}$
    c. $\frac{1}{2} - \frac{2}{3}x = \frac{1}{4}$
    d. $\frac{3}{4} - \frac{3}{5}x = \frac{19}{20}$
    e. $\frac{3}{2} = \frac{5}{6} + \frac{3x}{8}$

f. $\frac{11}{27} = \frac{4}{9} - \frac{2x}{3}$

24. Solve $2x - 3y = 8$ when $y = 0$.

25. If $2x - 3 = 7$, evaluate $3x + 4$.

26. ***Business Problems***

Cost =
Selling Price =
Markup =
Markup Rate =
Selling Price = Cost + Markup; Markup = Markup Rate (Cost)

$S = C + M; M = r \cdot C$

27. A watch costing $98 is sold for $156.80. Find the markup rate on the watch.

28. A pair of jeans with a selling price of $57 has a markup rate of 50%. Find the cost of the pair of jeans.

29. ***More Business Problems***

Sale Price =
Regular Price =
Discount =
Discount (Markdown) Rate =

Sale Price = Regular Price - Discount; Discount = Discount Rate(Regular Price)

$S = R - D;\ D = rR$

30. A DVD with a regular price of $20 is on sale for $15. Find the markdown rate.

31. A telescope is on sale for $165 after a markdown of 40% off the regular price. Find the regular price.

32. ***Physics.*** The distance s, in feet, that an object will fall in t seconds is given $s = 16t^2 + vt$, where v is the initial velocity of the object in feet per second. Find the initial velocity of an object that falls 80 feet in 2 seconds.

33. ***Goal:*** get the equation in the form *variable = constant.* Solve the following equations. Check your answers.

a. $4y + 3 = 11$
b. $8x + 5 = 4x + 13$
c. $12y - 3 = 15y - 9$
d. $2x - 3 = -11 - 2x$
e. $0.2b + 3 = 0.5b + 12$
f. $4y - 8 = y - 8$
g. $5 + 7x = 11 + 9x$
h. $8m = 3m + 20$
i. $3b + 5 = 8b - 5$
j. $2m - 1 = -6m + 5$

34. If $4x = 2x - 8$, evaluate $x^2 + 2x + 1$

35. If $x - 1 = 2x + 1$, evaluate $3x + 3$

36. Solve and check.
a. $5x + 2(x + 1) = 23$
b. $7a - (3a - 4) = 12$
c. $5y - 3 = 7 + 4(y - 2)$
d. $5[2 - 2(2x - 4)] = 2(5 - 3x)$
e. $5 + 3[1 + 2(x + 3)] = 4(x - 5)$

37. If $9 - 5x = 12 - (6x + 7)$, evaluate $x^2 - 3x - 2$.

38. Review: What is an equation?

39. Key Words that indicate "equals":
a. Equals
b. Is
c. Is equal to
d. Amounts to
e. Represents

40. Translate the sentence into an equation and solve.
a. The sum of six and a number is two. Find the number.
b. The difference between twelve and a number is five. Find the number.

c. Two-thirds of a number is negative twenty. Find the number.
d. The sum of twice a number and six is eighteen. Find the number.
e. Five times the sum of three times a number and two is forty-five.

41. Consecutive integers:
Consecutive even integers:
Consecutive odd integers:
    a. Find three consecutive integers whose sum is zero.
    b. Find two consecutive odd integers such that three times the first is one less than twice the second.

42. The sum of two numbers is twenty-five. The total of four times the smaller number and two is six less than the product of two and the larger number. Find the two numbers.

43. The sum of two numbers is eighteen. The total of three times the smaller and twice the larger is forty-four. Find the two numbers.

44. An isosceles triangle has two sides of equal length. The length of one of the equal sides is two more than three times the length of the third side. If the perimeter is 46 m, find the length of each side.

45. An investment of $8000 is divided into two accounts, one at a bank and the other at a credit union. The value of the investment at the credit union is $1000 less than twice the value at the bank. Find the amount in each account.

46. A 12-foot board is cut into two pieces. Twice the length of the shorter piece is 3 feet less than the length of the longer piece. Find the length of each piece.

47. The cellular phone service for a business executive is $35 per month plus $0.40 per minute of phone use. For a month in which the executive's cellular phone bill was $99.80, how many minutes did the executive use the phone? (**Note: The $35 is a base fee that does not change. It is called a fixed cost. The extra charges depend on how many minutes are used, $.40 per minute. This is called a variable cost. Fixed Cost + Variable Cost = Total Cost)

48. **Review:** A *set* is a collection of objects, which are called the *elements* of the set.

    Roster Method of Writing Sets: Encloses a list of the elements of the set in "set brackets."

Example: The set of cardinal directions would be written: {North, South, East, West}
Example: The set of prime numbers less than 10 is {2, 3, 5, 7}
Example: The set of integers between 0 and 10 is
Example: The set of natural numbers is

49. We can name a set. We often use a capital letter to denote a set.

Example: $P = \{2, 3, 5, 7\}$
Example: $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
Example: $N = \{1, 2, 3, 4, \dots\}$

50. The **empty set** or **null set** is the set that contains no elements. (Think empty basket.) Written in roster method, we have { }. We also use the symbol ∅. **Note: { } or ∅, never $\{\emptyset\}$.

51. ***Union of Sets.*** The union of the sets A and B, written $A \cup B$, is the set of all elements which belong to either set A **or** set B.
    a. Find $A \cup B$.
        i. $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$
        ii. $A = \{-3, -2, -1\}$ and $B = \{-2, -1, 0\}$
        iii. $A = \{a, b, c\}$ and $B = \{x, y, z\}$
        iv. $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$

52. ***Intersection of Sets.*** The intersection of the sets A and B, written $A \cap B$, is the set of elements which belong to both set A **and** set B.
    b. Find $A \cap B$.
        i. $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$
        ii. $A = \{-3, -2, -1\}$ and $B = \{-2, -1, 0\}$
        iii. $A = \{a, b, c\}$ and $B = \{x, y, z\}$
        iv. $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$

53. ***Set Builder Notation.*** In set builder notation, sets are written using a rule to describe the elements of the set.
    c. Example: The set of all integers less than 10 can be written

$$\{x | x < 10, x \in \text{integers}\}$$

**Note: The straight line, |, is read "such that"
**Note: The symbol ∈ is read "is in" or "is an element of"

The entire sentence reads "The set of all x values such that x is less than 10 and x is an element of the integers."

d. Example: The positive integers less than 5

$$\{x|x < 5, x \in \text{ positive integers}\}$$

Or

$$\{x|0 < x < 5, x \in \text{integers}\}$$

e. Use set builder notation to write the set.
   i. The integers less than -70.
   ii. The real numbers less than 15.
   iii. The real numbers between -2 and 8, excluding -2 and 8.
   iv. The real numbers between 6 and 12, including 6 but excluding 12.

Interval Notation. What is an interval?

The set $\{x|6 \leq x < 12\}$ can be written $[6, 12)$.
The bracket [means that 6 is included.
The parenthesis) means that 12 is not included.

The numbers 6 and 12 are the endpoints of the interval, and all the numbers between the endpoints are understood to be a part of the interval.

The interval $[6, 12)$ is read "the set of all real numbers between 6 and 12, including 6 but excluding 12."

**Note: To represent an interval that extends forever in the positive direction, we use the infinity symbol ∞. To represent an interval that extends forever in the negative direction, we use the negative infinity symbol $-\infty$.

**Note: We always use parentheses with the infinity symbols.

54. Write the set in interval notation.
   f. $\{x|-3 < x < 4\}$
   g. $\{x|-3 \leq x \leq 4\}$
   h. $\{x|-3 \leq x < 4\}$
   i. $\{x| - 3 < x \leq 4\}$
   j. $\{x|x < 4\}$

k. $\{x|x \leq 4\}$
l. $\{x|x > -3\}$
m. $\{x|x \geq -3\}$

55. Write the interval in set builder notation.
n. $(-1, 5)$
o. $(-1, 5]$
p. $[-1,5)$
q. $(-1, \infty)$
r. $(-\infty, 5]$

56. ***Graphing Sets.*** By graphing a set we mean to shade the numbers represented on the number line by the set.
s. Graph the set.
i. $(-3, 4)$
ii. $[-3, 4]$
iii. $(-3, 4]$
iv. $(-3, \infty)$
v. $(-\infty, 4]$
vi. $\{x|-1 < x \leq 5\}$
vii. $\{x|x < 5\}$
viii. $\{x|x \geq -1\}$

57. Review: What is a solution of an equation?
a. Solve the equation $3x - 2 = 4$.

58. What is a solution to an inequality?
a. Is 4 a solution to the inequality $3x - 2 \geq 4$? Is 5 a solution? Is 6 a solution? Is 1 a solution?

59. The **solution set** of an inequality is the set of all solutions to the inequality.
60. The solution set of $x > 3$ is:
61. The solution set of $x \leq 3$ is:
62. The goal is to get the inequality in the form $x < n$ or $x > n$.
63. Inequalities have similar properties that equations have:
a. The **Addition-Subtraction Property of Inequalities** states that adding or subtracting a real number to both sides of an inequality results in an equivalent inequality.
1. Example: $x + 5 > 8$
2. Example: $x + 5 \leq 8$
3. Example: $x - 4 < 12$
4. Example: $6x + 3 \geq 5x - 2$

5. Example: $2x - \frac{1}{2} < x + \frac{3}{4}$
6. Example: $3 - x < 0$

b. The **Multiplication-Division Property of Inequalities** has two parts (this is slightly different than the similar property for equations).

ii. Multiplying or dividing an inequality by a positive real number results in an equivalent inequality.

1. Example: $8x \leq 24$
2. Example: $5x \geq 0$
3. Example: $-2x < 0$

iii. When you multiply or divide by a negative real number, you need to reverse ("flip") the inequality symbol to

1. Example: $-2x < 0$
2. Example: $-3m < 12$
3. Example: $-8y \geq -16$

64. One-fourth of a number is less than five-sixths. Find the largest integer which satisfies this inequality.
65. In a person's daily diet, fat intake should be at most 30% of calorie intake. If a person who eats 2000 calories a day consumes 270 calories of fat at breakfast, how many more fat calories can this person consume during the rest of the day?
67. Solve the inequality.
    a. $3x - 4 \leq 5$
    b. $3 - 2y \geq 7$
    c. $6m - 2 > 4m$
    d. $5 - b < b + 3$
    e. $2n - 9 \geq 5n + 4$
    f. $3(x - 2) > 5x$
    g. $4(1 - x) > 2(x - 3)$
    h. $3 + 8(x - 5) < 3(x + 1)$

College of Southern Idaho Math Transition - Unit 4 Factoring

1. ***Review:*** Write the following numbers as a product of prime numbers.
   a. 42 b. 30 c. 24

2. ***Review:*** Find the greatest common factor.
   a. 24, 16 b. $8x, 12x$ c. $12x^2y, 15xy^3$

3. ***Review:*** Multiply. $3x(x^2 - 5)$

4. ***Factoring.*** In this chapter, we want to reverse the process of multiplication. That is, we want to be able to write a polynomial as a product of other polynomials (factors). This is called factoring.

5. ***Factoring out the GCF.***
   a. Find the GCF of the terms of the polynomial.
   b. Rewrite each term of the polynomial as a product of the GCF and some other term.
   c. Use the Distributive Property to write the polynomial as a product of factors.
6. Factor:
   a. $8x + 12$
   b. $6x^2 - 15x$
   c. $9xy + 18x^2$
   d. $10y^2 - 15xy^3$
   e. $12m^2 + 6m - 18$
   f. $20x^4y^3 - 30x^3y^4 + 40x^2y^5$
   g. $x(x-3) + 6(x-3)$

7. ***Review.*** Multiply: $(x+6)(x-3)$

8. Factor.
   a. $a(x-y) + b(x-y)$
   b. $6x(4x+3) - 5(4x+3)$
   c. $3w(5w-1) + 2(1-5w)$

9. ***Factoring by Grouping.*** To factor a polynomial by grouping,
   a. Group terms together (Usually start with first two and last two terms)
   b. Factor each group.
   c. Factor the GCF from each group.

10. Factor.
    a. $x^2 + xy + 2x + 2y$
    b. $xy - y^2 + 2x - 2y8x^2 - 12x - 6xy + 9y$
    c. $7xy^2 - 3y + 14xy - 6$
    d. $5xy - 9y + 10x - 18$
    e. $ab + 6b - 4a - 24$
11. ***Factoring trinomials.*** To factor a polynomial of the form $x^2 + bx + c$:
    a. Find two terms, p and q, whose product is c and whose sum is b
    b. $x^2 + bx + c = (x + p)(x + q)$

Note: p and q can be negative

12. Factor:
    a. $x^2 + 4x + 3$
    b. $x^2 + 3x - 10$
    c. $x^2 - 8x - 20$
    d. $x^2 - 8x + 15$
    e. $x^2 - 9$
13. ***Factoring Completely.*** A polynomial is factored completely if it cannot be factored any further over the integers. To factor completely:
    a. Factor the GCF.
    b. Factor the trinomial.
14. Factor Completely:
    a. $3x^2 - 6x - 72$
    b. $2x^3 + 6x^2 - 20x$
    c. $-4y^3 + 28y^2 - 48y$
    d. $3x^2y - 6xy - 45y$
    e. $x^2 + 6xy + 5y^2$
    f. $5x^2 - 10xy - 15y^2$

15. ***Factoring.*** ($a \cdot c$ Method or Grouping Method) To factor a polynomial of the form $ax^2 + bx + c$:

a. Find the value of $a \cdot c$

b. Find two terms, p and q, such that $p + q = b$ and $p \cdot q = a \cdot c$

c. Rewrite: $ax^2 + bx + c = ax^2 + px + qx + c$

d. Factor this expression by grouping.

Note: p and q can be negative

16. Factor:

a. $x^2 + 4x + 3$

b. $2x^2 - 3x - 5$

c. $6x^2 + x - 2$

d. $3x^2 + 7x + 4$

e. $15x^2 - 50x + 35$

f. $2x^3 - 11x^2 + 5x$

g. $3x^2 + 5xy - 2y^2$

h. $4x^3y - 4x^2y^2 + xy^3$

17. ***Factoring.*** (Guess and Check Method) To factor a polynomial of the form $ax^2 + bx + c$:

a. Find two terms whose product is a: m and n

b. Find two terms whose product is c: p and q

c. Multiply: $(mx + p)(nx + q)$

i. If $(mx + p)(nx + q) = ax^2 + bx + c$, you are done.

ii. If $(mx + p)(nx + q) \neq ax^2 + bx + c$, repeat the process with other pairs.

d. Note:
    i. If c is positive and b is positive, then p and q are both positive.
    ii. If c is positive and b is negative, then p and q are both negative.
    iii. If c is negative, then p and q have opposite signs.

18. Factor:
    a. $5x^2 - 2x - 3$
    b. $-12x^3 - 18x^2 + 30x$

19. ***Difference of squares.*** $a^2 - b^2 = (a + b)(a - b)$
    a. Factor.
        i. $4x^2 - 9$
        ii. $25x^2 - 16y^2$
        iii. $36x^3 - 100x$

20. ***Review.*** Factor:
    a. $x^2 + 2x + 1$
    b. $x^2 - 4x + 4$

21. ***Perfect Square Trinomials.*** Expand:
    a. $(a + b)^2$
    b. $(a - b)^2$

22. Factor:
    a. $4x^2 + 4x + 1$
    b. $100x^2 - 180x + 81$
    c. $9x^2 + 15x + 4$

23. ***General Strategy for Factoring.***
    a. Look for GCF.
    b. Look for Special Factoring Forms.
        i. Difference of Squares.
        ii. Perfect Square Trinomial.
    c. Use methods of factoring quadratic polynomials: $ax^2 + bx + c$
    d. Determine if you can factor by grouping.
    e.

24. Factor the polynomial completely.
    a. $9x^2 - 81$
    b. $4x^3 + 28x^2 - 120x$
    c. $9x^2y - 48xy + 48y$
    d. $a^2b - 3a^2 - 16b + 48$

25. ***Review:*** Multiply.

a. $3 \cdot 0$ b. $0 \cdot 3$ c. $a \cdot 0$ d. $0 \cdot a$

26. ***Principle of Zero Products.*** If the product of two numbers is zero, then at least one of the numbers must be zero.

If $ab = 0$, then $a = 0$ or $b = 0$. (Both a and b could be 0.)

27. How do we use this fact?

28. Solve the equation.

    a. $(x - 2)(x + 5) = 0$

    b. $3x(x - 4) = 0$

29. ***Solving Equations by Factoring.***

    a. Rewrite the equation, if necessary, so that 0 is on one side of the equation.
    b. Factor the polynomial.
    c. Use the Principle of Zero Products.
    d. Solve each equation.

30. Solve the equation.

    a. $x^2 - 16 = 0$
    b. $25x^2 - 16 = 0$
    c. $x^2 - 8x + 15 = 0$
    d. $y^2 + y - 30 = 0$
    e. $y^2 - 7y = 8$
    f. $3t^2 + t = 10$
    g. $x(x - 11) = 12$
    h. $(x + 4)(x - 1) = 14$

31. The sum of the squares of two consecutive positive integers is forty-one. Find the two integers.

32. The length of a rectangle is 5 inches more than twice its width. Its area is 75 square inches. Find the length and width of the rectangle.

College of Southern Idaho Math Transition - Unit 5 Rational Expressions

1. ***Rational Expressions.*** A fraction in which the numerator and denominator are polynomials is called a rational expression.

   a. Examples:

      i. $\frac{1}{x}$

      ii. $\frac{x+1}{x^2+2x+1}$

2. ***Simplifying Rational Expressions.*** Just like with other fractions, a rational expression is simplified if the numerator and denominator **have no common factors**. To simplify a rational expression:

   a. Completely factor the numerator and denominator.

   b. Divide by common factors. (Common factors divide to 1, not 0)

3. Simplify.

   a. $\frac{9x^3}{12x^4}$

   b. $\frac{2x-1}{1-2x}$

   c. $\frac{x+1}{x^2+2x+1}$

   d. $\frac{x^2-1}{x^2+4x-5}$

   e. $\frac{a^2-2a}{4-2a}$

4. ***Review:*** Multiply.

   a. $\frac{1}{2} \cdot \frac{3}{5}$

   b. $\frac{2}{3} \cdot \frac{3}{4}$

   c. $\frac{6}{7} \cdot \frac{5}{9}$

5. ***Multiplying Rational Expressions.*** Just like with fractions, when we multiply rational expressions we want the result to be in simplest form. To multiply rational expressions:

   a. Factor each numerator and each denominator.

   b. Multiply. (Numerators by numerators, denominators by denominators)

   c. Divide by common factors. (Common factors divide to 1, not 0)

6. Multiply.

   a. $\frac{9x^5}{10y^2} \cdot \frac{5y^5}{12x^3}$

   b. $\frac{6x^2-10x}{3x-3} \cdot \frac{x^2-1}{12x-20}$

   c. $\frac{x^2-4}{x^2+5x+6} \cdot \frac{x^2+6x+9}{x^2-2x}$

   d. $\frac{x^2-8x+7}{x^2+3x-4} \cdot \frac{x^2+3x-10}{x^2-9x+14}$

   e. $\frac{y^2+y-20}{y^2+2y-15} \cdot \frac{y^2+4y-21}{y^2+3y-28}$

7. ***Review:*** Divide.

   a. $\frac{1}{2} \div \frac{3}{5}$

   b. $\frac{2}{3} \div \frac{4}{9}$

8. ***Dividing Rational Expressions.*** Just like other fractions, to divide a rational expression by another rational expression multiply the dividend by the reciprocal of the divisor.

9. Divide.

   a. $\frac{6x-12}{8x+32} \div \frac{18x-36}{10x+40}$

   b. $\frac{x^2-5x+6}{x^2-9x+18} \div \frac{x^2-6x+8}{x^2-9x+20}$

   c. $\frac{x^2+2x-15}{x^2-4x-45} \div \frac{x^2+x-12}{x^2-5x-36}$

   d. $\frac{4x^2y^3}{15a^2b^3} \div \frac{6xy}{5a^3b^5}$

10. ***Adding or Subtracting Rational Expressions with the Same Denominator.*** If two rational expressions have the same denominator, add or subtract by adding or subtracting the numerators. Remember to simplify.

11. Add or subtract.

    a. $\frac{18}{5x}+\frac{7}{5x}$

    b. $\frac{x}{x^2-1}+\frac{1}{x^2-1}$

    c. $\frac{x}{x^2+2x-15}-\frac{3}{x^2+2x-15}$

    d. $\frac{2x+3}{x^2-x-30}-\frac{x-2}{x^2-x-30}$

    e. $\frac{x-7}{2x+7}-\frac{4x-3}{2x+7}$

    f. $\frac{2x^2+3x}{x^2-9x+20}+\frac{2x^2-3}{x^2-9x+20}-\frac{4x^2+2x+1}{x^2-9x+20}$

12. ***Review:*** Add or subtract.

    a. $\frac{1}{2}+\frac{1}{3}$

    b. $\frac{1}{2}-\frac{1}{4}$

13. ***Review:*** Find the Least Common Multiple (LCM).

    a. 3, 4

    b. 6, 9

    c. $12x^2, 18x$

    d. $6x^2y, 9xy^3$

    e. $x^2-1, x+1$

    f. x^2+4x+4, x^2-4

    g. $x^2+5x+4, x^2-3x-28$

14. ***Adding and Subtracting Rational Expressions with Different Denominators.*** To add or subtract rational expressions with different denominators

a. Find the Least Common Denominator, which is the LCM of the denominators.

b. Rewrite each fraction with the Least Common Denominator by multiplying the numerator and denominator by the missing factors.

c. Now each fraction should have the same denominator. Add or subtract as before.

15. Add or Subtract.

a. $\frac{x-3}{x^2-2x}+\frac{6}{x^2-4}$

b. $\frac{x-2}{10x}-\frac{x-3}{15x}$

c. $\frac{4}{x-2}+\frac{5}{x+3}$

d. $\frac{x}{x^2-9}+\frac{3}{x-3}$

e. $\frac{7}{x+2}+2$

f. $\frac{a-1}{a^2b}-\frac{a-2}{ab}$

g. $\frac{2x}{3x-2}+\frac{4}{x-3}$

h. $\frac{5x}{x-2}+\frac{3}{2-x}$

i. $\frac{6y}{y^2-4}+\frac{3}{2-y}$

j. $\frac{2a+3}{a^2-7a+12}-\frac{2}{a-3}$

16. ***Complex Fractions.*** A complex fraction is a fraction in which the numerator and/or the denominator contain one or more fractions.

a. Examples: $\frac{2}{1+\frac{1}{3}}, \frac{1+\frac{1}{x}}{2}, \frac{\frac{4}{y}}{6-\frac{3}{y}}$

b. To simplify a complex fraction, we want to find an equivalent expression which does not have fractions in the numerator or the denominator.

17. ***Review.*** Multiply

a. $\frac{x+1}{x} \cdot 2x$

b. $\frac{x-1}{2x} \cdot 2x$

18. ***Simplifying Complex Fractions (Method 1).***

    a. Find the Least Common Denominator of the fractions in the numerator and denominator of the complex fraction.

    b. Multiply the numerator and Denominator of the complex fraction by the LCD.

    c. Simplify.

19. Use method 1 to simplify the complex fraction: $\frac{\frac{x+1}{x}}{\frac{x-1}{2x}}$

20. ***Simplifying Complex Fractions (Method 2).***

    a. Simplify the numerator to a single fraction and simplify the denominator to a single fraction.

    b. Divide the numerator by the denominator. (Multiply the numerator by the reciprocal of the denominator.)

    c. Simplify.

21. Use method 2 to simplify the complex fraction: $\frac{\frac{x+1}{x}}{\frac{x-1}{2x}}$

22. Simplify.

    a. $\frac{2+\frac{1}{y}}{3-\frac{2}{y}}$

    b. $\frac{1+\frac{4}{x}}{1-\frac{16}{x^2}}$

    c. $\frac{a-\frac{10}{a-3}}{1+\frac{5}{a-3}}$

    d. $\frac{1+\frac{4}{x}+\frac{4}{x^2}}{1-\frac{2}{x}-\frac{8}{x^2}}$

    e. $\frac{a+4+\frac{5}{a-2}}{a+6+\frac{15}{a-2}}$

f. $\frac{\frac{7}{x-3}-\frac{2}{3x}}{\frac{5}{3x}+\frac{1}{x-3}}$

23. ***Review.*** Solve the equation.

a. $\frac{2}{3}x+3=\frac{7}{2}$

b. $\frac{x}{3}-\frac{1}{4}=\frac{1}{12}$

c. $\frac{3x+4}{12}-\frac{1}{3}=\frac{5x+2}{12}-\frac{1}{2}$

24. To solve an equation containing fractions:

a. If the variable is in a denominator, find the values of the variable that would result in division by zero. (A solution cannot be one of these numbers.)

b. Multiply both sides of the equation by the Least Common Denominator.

c. Solve the equation.

25. Solve the equation.

a. $\frac{12}{x-5}=2$

b. $5-\frac{8}{x}=1$

c. $\frac{5}{x+3}=\frac{3}{x-1}$

d. $3-\frac{4}{x+2}=\frac{2x}{x+2}$

e. $\frac{x}{x+12}=\frac{1}{x+5}$

f. $\frac{3x}{2x+1}+\frac{1}{x+2}=\frac{4}{x+2}$

26. Solve the proportion:

a. $\frac{x}{15} = \frac{3}{5}$

b. $\frac{10}{x} = \frac{5}{7}$

c. $\frac{3}{8} = \frac{12}{x-2}$

d. $\frac{9}{x+2} = \frac{3}{x-2}$

e. $\frac{16}{x-2} = \frac{8}{x}$

27. ***Using Proportions to Solve Applications.***
    a. The monthly loan payment for a car is $28.35 for each $1000 borrowed. At this rate, find the monthly payment for a $6000 car loan.

    b. Biologist catch, tag, and release 125 trout in a pond. Later, 200 trout are caught and checked for tags. Forty of these trout are found to have tags. Estimate the total trout population of the pond.

    c. An exit poll survey showed the 5 out of every 8 voters cast a ballot in favor of an amendment to a city charter. At this rate, how many voters voted in favor of the amendment if 40,000 people voted?

28. **Solve for the indicated variable.**
    a. Solve $A = lw$ for w.
    b. Solve $2x + y = 3$ for y.
    c. Solve $3x - 4y = 7$ for y.
    d. Solve $y - 4 = 2(x + 3)$ for y.
    e. Solve $x - 3y = 4$ for x.
    f. Solve $x + 2y - 1 = 0$ for x.
    g. Solve $P = 2l + 2w$ for w.
    h. Solve $P = R - C$ for R.
    i. Solve $S = C - rC$ for C.
    j. Solve $S = rS + C$ for S.

1. ***The Cartesian Plane.***

2. ***Ordered Pairs.*** For the ordered pair $(1, 2)$, the first number, in this case 1, is the x-coordinate and the second number, 2, is the y-coordinate. **Order matters**. For example, the ordered pairs $(1, 2)$ and $(2, 1)$ represent two different points.

   a. Graph the ordered pairs in the Cartesian coordinate system.
      i. $(1,2)$
      ii. $(2, 1)$
      iii. $(-1, 2)$
      iv. $(2, -1)$
      v. $(-1, -2)$
      vi. $(0, 0)$

Solutions of an equation in two variables. A solution of an equation in two variables is an ordered pair (x, y) whose coordinates make the equation a true statement.

   b. Example: $(1, 2)$ is a solution of $y = x + 1$ because when we replace x with 1 and y with 2 we get a true statement: $2 = 1 + 1$

      i. Note: **Order Matters!** $(2,1)$ is *not* a solution to $y = x + 1$

   c. Is $(0, -1)$ a solution of $2x - y = 1$?

   d. Is $(2, -2)$ a solution of $y = 3x + 8$?

   e. Graph the ordered-pair solutions of $y = -3x$ when $x = -1, 0$, and 1.

3. ***Relations.*** A relation is something that connects two or more different things together. In mathematics, a relation is any set of ordered pairs.

   a. Example. $\{(1, 2), (3, 4), (1, 4)\}$ is a relation.

   b. Example. $\{(-2, 4), (3, 8), (6, -5), (1, 9)\}$ is a relation

4. ***Domain and Range.*** The domain of a relation is the set of first coordinates. The range of a relation is the set of second coordinates.

   a. Find the domain and range:
      i. $\{(1, 2), (3, 4), (1, 4)\}$
      ii. $\{(-2, 4), (3, 8), (6, -5), (1, 9)\}$

5. ***Functions.*** A function is a special type of relation. A function is a relation in which no two ordered pairs have the same first coordinate (*x*-coordinate) and different second coordinates (*y*-coordinates).

   a. Example. $\{(1,2),(3,4),(1,4)\}$ is **not** a function. Why?

   b. Does the relation $\{(-2,4),(3,8),(6,-5),(1,9)\}$ define a function?

6. We say that "an equation defines y as a function of x" if for each x-value there is only one corresponding y-value. That is, y is a function of x if the set of ordered pair solutions is a function.

   a. Does $y = 2x - 3$ where $x \in \{-2,-1,0,1,2\}$ define y as a function of x?

   b. Does $|y| = x$ where $x \in \{1,2,3,4\}$ define y as a function of x?

   c. Does $y = x^2$, where $x \in \{-2,-1,0,1,2\}$ define y as a function of x?

7. ***Function Notation.*** If y is a function of x, we replace y with $f(x)$, read "f of x," to emphasize that the relation is a function.

   a. Example: Since $y = 2x - 3$ defined y as a function of x, we write $f(x) = 2x - 3$.

   b. The symbol $f(x)$ does not mean f times x. It simply means the value of the function at x.

      i. Example $f(4)$ means find the value of the function when $x = 4$.

   c. For $f(x) = 2x - 3$, find $f(4)$.

   d. For $f(x) = \frac{x}{x-2}$, find $f(3)$.

   e. For $G(x) = x^2 + x$, find $G(-1)$.

   f. For $s(t) = \frac{5}{t-2}$, find $s(7)$.

8. ***Graphs of Equations in two variables.*** The graph of an equation in two variables is the graph of the ordered-pair solutions of the equation.

9. ***Graphs of Linear Equations in Two Variables.*** The graph of a linear equation in two variables, $y = mx + b$, is a straight line.

10. Graph the Equation.

   a. $y = 2x + 1$

   b. $y = x - 3$

   c. $y = -x + 1$

   d. $y = \frac{1}{2}x$

   e. $y = -\frac{1}{4}x + 1$

11. ***Standard Form of a Linear Equation in Two Variables.*** The standard form of a linear equation in two variables is $Ax + By = C$, e.g. $2x + y = 3$.

   a. To graph a linear equation in two variables written in standard form, first solve for y.

12. Graph the equation.

   a. $2x + y = 3$

   b. $4x - 2y = 6$

13. ***Horizontal Lines.***

   a. Graph $y = 3$

   b. Graph $y = -2$

14. ***Vertical Lines.***

   a. Graph $x = 3$

   b. Graph $x = -2$

15. ***x-intercepts:*** An x-intercept of a graph is a point at which the graph crosses the x-axis:

a. The y-coordinate of an x-intercept is always 0.

b. To find the x-intercept, set $y = 0$ and solve for x.

c. CAUTION: The x-intercept is a point, so it should be written as an ordered pair.

16. Find the x-intercept of the graph of the equation.

a. $2x - 3y = 12$

b. $x - 2y = 0$

c. $y = \frac{1}{3}x - 3$

d. $2x - 5y = 5$

e. $4x + 3y = -12$

17. ***y-intercepts:*** A y-intercept of a graph is a point at which the graph crosses the y-axis:

a. The x-coordinate of a y-intercept is always 0.

b. To find the y-intercept, set $x = 0$ and solve for y.

c. CAUTION: The y-intercept is a point, so it should be written as an ordered pair.

18. Find the x-intercept of the graph of the equation.

a. $2x - 3y = 12$

b. $x - 2y = 0$

c. $y = \frac{1}{3}x - 3$

d. $2x - 5y = 5$

e. $4x + 3y = -12$

19. ***Slope of a Straight Line.*** Lines rise or fall at a constant rate. This rate is called the slope of the line.

$$\text{Slope} = m = \frac{\text{rise}}{\text{run}}$$

20. ***Slope Formula.*** If P_1 is the point (x_1, y_1) and P_2 is the point (x_2, y_2), then the slope of the line that passes through P_1 and P_2 is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \qquad x_1 \neq x_2$$

If $x_1 = x_2$, the slope of the line is undefined.

21. Find the slope of the line containing the given points. Graph the line.

    a. $P_1(2,1),\ P_2(3,4)$
    b. $P_1(5,3),\ P_2(-2,5)$
    c. $P_1(4,3),\ P_2(-1,3)$
    d. $P_1(2,-2),\ P_2(2,4)$

22. ***Parallel Lines.*** Two non-vertical lines in the plane are parallel if and only if they have the same slope. Vertical lines in a plane are parallel to each other.

23. ***Perpendicular Lines.*** Two non-vertical lines in the plane are perpendicular if and only if the product of their slopes is -1, that is to say, if and only if their slopes are negative reciprocals. A vertical line is perpendicular to a horizontal line in the plane.

24. Determine whether the line through P_1 and P_2 is parallel, perpendicular, or neither parallel nor perpendicular to the line through Q_1 and Q_2.

    a. $P_1(1,3),\ P_2(2,6);\ \ Q_1(1,4),\ Q_2(2,7)$
    b. $P_1(1,3),\ P_2(2,6);\ \ Q_1(3,1),\ Q_2(6,2)$
    c. $P_1(1,3),\ P_2(2,6);\ \ Q_1(3,-1),\ Q_2(6,-2)$

25. Find the slope and y-intercept of the line $y = 3x + 2$.

26. ***Slope-intercept form of a Linear Equation.*** An equation of the form $y = mx + b$ is the equation of a line whose slope is m and whose y-intercept is $(0, b)$.

    a. Find the slope and y-intercept of the graph.

        i. $y = 2x + 1$
        ii. $y = -\frac{2}{3}x + 3$
        iii. $2x - 3y = 5$

    b. Graph by using the slope and y-intercept.

        i. $y = 2x + 1$
        ii. $y = \frac{3}{2}x - 3$
        iii. $3x - 4y = 12$