



## Contents

Characteristics of Parallel Circuits.....	4
Voltage in a Parallel Circuit.....	5
Knowledge Check.....	7
Current in a Parallel Circuits .....	7
Knowledge Check.....	15
Resistance in a Parallel Circuit.....	16
Conductance .....	16
Two Equal Resistors in Parallel .....	19
Two Unequal Resistors in Parallel .....	21
Reciprocal Method .....	24
Product over Sum Method .....	25
Knowledge Check.....	26
Embedded Media .....	27
Power in a Parallel Circuit .....	27
Knowledge Check .....	29
Rules for Parallel DC Circuits .....	29
Solving Parallel Circuit Problems .....	29
Equivalent Circuits .....	30
Knowledge Check .....	38
Troubleshooting Parallel Circuits.....	40
Embedded Media .....	43
Parallel Circuits Lab 1: Simple Parallel Circuit.....	44
How to wire a breadboard in Parallel .....	44
Components & Equipment Needed .....	45
Circuit Diagram.....	45
Procedure .....	45
Table for Parallel Circuits Lab 1 .....	46
Observations and Conclusions .....	46
Parallel Circuits Lab 2: Solving a Parallel Circuit and Reducing to its Equivalent Circuit .....	47
Components & Equipment Needed .....	47
Circuit Diagram.....	47
Procedure .....	47
Tables for Parallel Circuits Lab 2 .....	48

## Air Washington Electronics – Direct Current

Observations and Conclusions .....	48
Parallel Circuits Lab 3: Critical Thinking .....	49
The Problem .....	49
Problem Solving Help.....	50
Steps to Solving.....	50
Answers to Knowledge Checks .....	51
Voltage in a Parallel Circuit .....	51
Current in a Parallel Circuit .....	51
Resistance in a Parallel Circuit.....	51
Conductance.....	52
Power in a Parallel Circuit .....	52
Solving Parallel Circuit Problems.....	52
Additional Resources .....	55
Physics Resources.....	55
Video Resources .....	55
References .....	56
Attributions .....	57
Table of Figures.....	58
Table of Tables .....	59
Table of Circuits .....	59
Table of Equations .....	59

## Characteristics of Parallel Circuits

The discussion of electrical circuits presented up to this point has been concerned with series circuits in which there is only one path for current. There is another basic type of circuit known as the parallel circuit with which you must become familiar. Where the series circuit has only one path for current, the parallel circuit has more than one path for current.

Ohm's law and Kirchhoff's law apply to all electrical circuits, but the characteristics of a parallel dc circuit are different than those of a series dc circuit.

A parallel circuit is defined as one having more than one current path connected to a common voltage source. Parallel circuits, therefore, must contain two or more resistances which are not connected in series. An example of a basic parallel circuit is shown in

Figure 1.

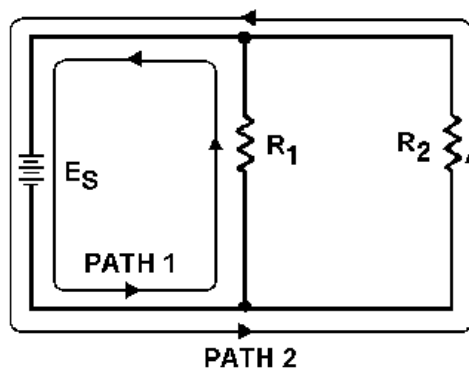


Figure 1: Example of a basic parallel circuit.

Recall from earlier modules that  $E$  (electromotive force) and  $V$  (voltage) are often used interchangeably when referring to a voltage source. Start at the voltage source ( $E_s$ ) and trace

counterclockwise around the circuit. Two complete and separate paths can be identified in which current can flow. One path is traced from the source, through resistance  $R_1$ , and back to the source. The other path is from the source, through resistance  $R_2$ , and back to the source.

### ***Voltage in a Parallel Circuit***

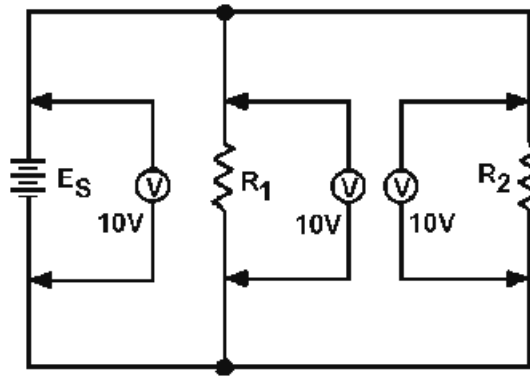
In the previous module, you learned that series circuits acted as voltage dividers: the source voltage in a series circuit divides proportionately across each resistor in the circuit. In a parallel circuit, however, the same voltage is present in each **branch**. A branch is a section of a circuit that has a complete path for current. In

$$E_S = E_{R1} = E_{R2}$$

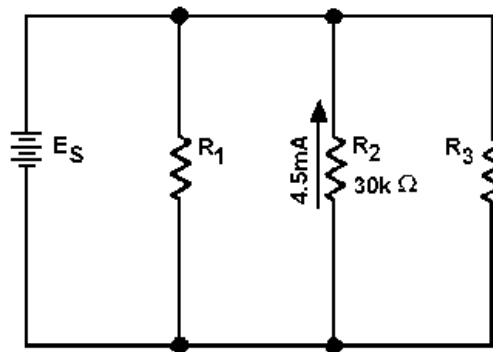
Figure 1 this voltage is equal to the applied voltage ( $E_s$ ). This can be expressed in equation form as:

**Equation 1: Voltage in a parallel circuit.**

Voltage measurements taken across the resistors of a parallel circuit, as illustrated in Figure 2 verify this equation. Each meter indicates the same amount of voltage. Notice that the voltage across each resistor is the same as the applied voltage.



**Figure 2: Voltage comparison in a parallel circuit.**



**Figure 3: Example problem parallel circuit.**

**Example:** Assume that the current through a resistor of a parallel circuit is known to be 4.5 milliamperes (4.5 mA) and the value of the resistor is 30,000 ohms (30 kΩ). Determine the source voltage. The circuit is shown in Figure 3.

**Given:**  
 $R_2 = 30,000 \text{ ohms (30k}\Omega\text{)}$   
 $I_{R2} = 4.5 \text{ milliamps (4.5mA or .0045 amps)}$

**Solution:**  
 $E = IR$   
 $E_{R2} = .0045 \text{ amp} \times 30,000 \text{ ohms}$   
 $E_{R2} = 135 \text{ volts}$

Since the source voltage is equal to the voltage of a branch:

$$E_S = E_{R2}$$

$$E_S = 135 \text{ volts}$$

To simplify the math operation, the values can be expressed in powers of ten as follows:

$$30,000 \text{ ohms} = 30 \times 10^3 \text{ ohms}$$

$$4.5\text{mA} = 4.5 \times 10^{-3} \text{ amps}$$

$$E_{R2} = (4.5 \times 10^{-3}) \text{ amps} \times (30 \times 10^3) \text{ ohms}$$

$$E_{R2} = (4.5 \times 30 \times 10^{-3} \times 10^3) \text{ volts}$$

$$(10^{-3} \times 10^3 = 10^{-3+3} = 10^0 = 1)$$

$$E_{R2} = (4.5 \times 30 \times 1) \text{ volts}$$

$$E_{R2} = 135 \text{ volts}$$

$$E_s = E_{R2}$$

$$E_s = 135 \text{ volts}$$

### Knowledge Check

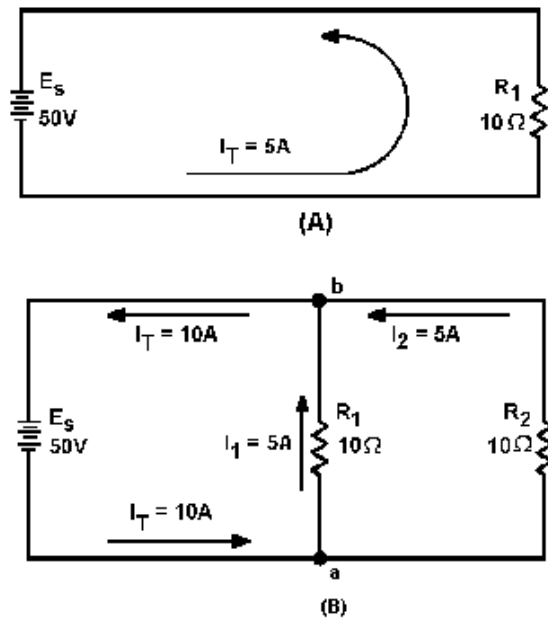
1. What would the source voltage ( $E_s$ ) in Figure 3 be if the current through  $R_2$  were 2 milliamps?

### *Current in a Parallel Circuits*

Ohm's law states that the current in a circuit is inversely proportional to the circuit resistance. This fact is true in both series and parallel circuits. Unlike a series circuit, which is a voltage divider, a parallel circuit acts as a current divider. There is a single path for current in a series circuit. The amount of current is determined by the total resistance of the circuit and the applied voltage. In a parallel circuit the source current divides among the available paths.

The behavior of current in parallel circuits will be shown by a series of illustrations using example circuits with different values of resistance for a given value of applied voltage.





**Figure 4: Analysis of current in parallel circuit.**

Part (A) shows a basic series circuit. Here, the total current must pass through the single resistor. The amount of current can be determined.

**Given:**  $E_s = 50V$  and  $R_1 = 10\Omega$

**Solution:**

$$I = \frac{E}{R}$$

$$I_T = \frac{E_s}{R_1}$$

$$I_T = \frac{50 \text{ volts}}{10 \text{ ohms}}$$

$$I_T = 5 \text{ amps}$$

Part (B) of

Figure 4 shows the same resistor ( $R_1$ ) with a second resistor ( $R_2$ ) of equal value connected in parallel across the voltage source. When Ohm's law is applied, the current flow through each resistor is found to be the same as the current through the single resistor in part (A).

**Given:**

$$E_s = 50 \text{ volts}$$

$$R_1 = 10 \text{ ohms}$$

$$R_2 = 10 \text{ ohms}$$

**Solution:**

$$I = \frac{E}{R}$$

$$E_s = E_{R1} = E_{R2}$$

$$I_{R1} = \frac{E_{R1}}{R_1}$$

$$I_{R1} = \frac{50 \text{ volts}}{10 \text{ ohms}}$$

$$I_{R1} = 5 \text{ amps}$$

$$I_{R2} = \frac{E_{R2}}{R_2}$$

$$I_{R2} = \frac{50 \text{ volts}}{10 \text{ ohms}}$$

$$I_{R2} = 5 \text{ amps}$$

It is apparent that if there is 5 amperes of current through each of the two resistors, there must be a total current of 10 amperes drawn from the source.

The total current of 10 amperes, as illustrated in

Figure 4(B), leaves the negative terminal of the battery and flows to *point a*. Since *point a* is a connecting point for the two resistors, it is called a **node** or **junction**. At *node a*, the total current divides into two currents of 5 amperes each. These two currents flow through their respective resistors and rejoin at *node b*. The total current then flows from *node b* back to the positive terminal of the source. The source supplies a total current of 10 amperes and each of the two equal resistors carries one-half the total current.

Each individual current path in the circuit of

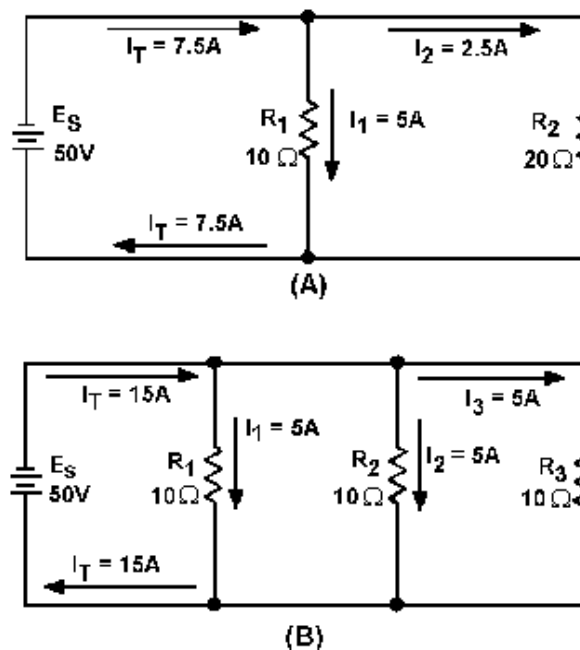
Figure 4(B) is referred to as a **branch**. Each branch carries a current that is a portion of the total current. Two or more branches form a **network**. From the previous explanation, the characteristics of current in a parallel circuit can be expressed in terms of the following general equation:

$$I_T = I_1 + I_2 + \dots I_n$$

**Equation 2: Current in a parallel circuit.**

Compare part (A) of Figure 5 with part (B) of the circuit in

Figure 4. Notice that doubling the value of the second branch resistor ( $R_2$ ) has no effect on the current in the first branch ( $I_{R1}$ ), but does reduce the second branch current ( $I_{R2}$ ) to one-half its original value. The total circuit current drops to a value equal to the sum of the branch currents. These facts are verified by the following equations.



**Figure 5: Current behavior in parallel circuits.**

**Given:**

$$\begin{aligned}E_s &= 50 \text{ volts} \\R_1 &= 10 \text{ ohms} \\R_2 &= 20 \text{ ohms}\end{aligned}$$

**Solution:**

$$I = \frac{E}{R}$$

$$E_s = E_{R1} = E_{R2}$$

$$I = \frac{E_{R1}}{R_1}$$

$$I = \frac{50 \text{ volts}}{10 \text{ ohms}}$$

$$I_{R1} = 5 \text{ amps}$$

$$I_{R2} = \frac{E_{R2}}{R_2}$$

$$I_{R2} = \frac{50 \text{ volts}}{20 \text{ ohms}}$$

$$I_{R2} = 2.5 \text{ amps}$$

$$I_T = I_{R1} + I_{R2}$$

$$I_T = 5 \text{ amps} + 2.5 \text{ amps}$$

$$I_T = 7.5 \text{ amps}$$

The amount of current flow in the branch circuits and the total current in the circuit shown in Figure 5(B) are determined by the following computations.

**Given:**  
 $E_s = 50 \text{ volts}$   
 $R_1 = 10 \text{ ohms}$   
 $R_2 = 10 \text{ ohms}$   
 $R_3 = 10 \text{ ohms}$

**Solution:**  $I = \frac{E}{R}$

$$E_s = E_{R1} = E_{R2} = E_{R3}$$

$$I_{R1} = \frac{E_{R1}}{R_1}$$

$$I_{R1} = \frac{50 \text{ volts}}{10 \text{ ohms}}$$

$$I_{R1} = 5 \text{ amps}$$

$$I_{R2} = \frac{E_{R2}}{R_2}$$

$$I_{R2} = \frac{50 \text{ volts}}{10 \text{ ohms}}$$

$$I_{R2} = 5 \text{ amps}$$

$$I_{R3} = \frac{E_{R3}}{R_3}$$

$$I_{R3} = \frac{50 \text{ volts}}{10 \text{ ohms}}$$

$$I_{R3} = 5 \text{ amps}$$

$$I_T = I_{R1} + I_{R2} + I_{R3}$$

$$I_T = 5 \text{ amps} + 5 \text{ amps} + 5 \text{ amps}$$

$$I_T = 15 \text{ amps}$$

Notice that the sum of the ohmic values in each circuit shown in Figure 5 equals 30 ohms, and that the applied voltage is the same (50 volts). However, the total current in Figure 5(B) (15

amps) is twice the amount in Figure 5(A) (7.5 amps). It is apparent, therefore, that the manner in which resistors are connected in a circuit, as well as their actual ohmic values, affect the total current.

The division of current in a parallel network follows a definite pattern. This pattern is described by **Kirchhoff's current law** which states:

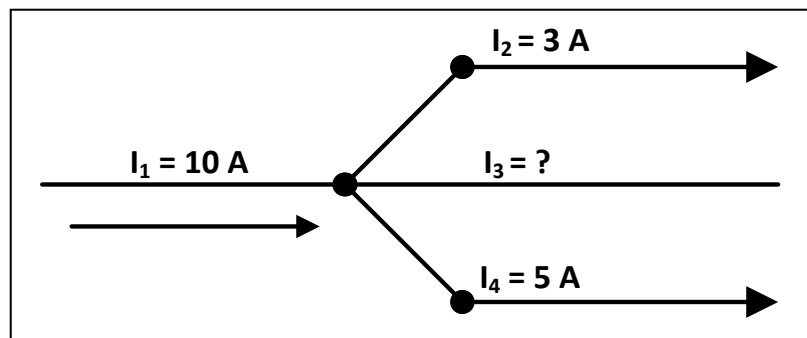
*The algebraic sum of the currents entering and leaving any junction of conductors is equal to zero.*

This law can be stated mathematically as:

$$I_a + I_b + \dots + I_n = 0$$

**Equation 3: Kirchhoff's Current Law**

$I_a$ ,  $I_b$ , etc., are the currents entering and leaving the junction. Currents entering the junction are considered to be positive and currents leaving the junction are considered to be negative. When solving a problem using Kirchhoff's current law, the currents must be placed into the equation **with the proper polarity signs attached**.



**Figure 6: Circuit for example problem.**

**Example:** Solve for the value of  $I_3$  in Figure 6.

**Given:**  
 $I_1 = 10 \text{ amps}$   
 $I_2 = 3 \text{ amps}$   
 $I_4 = 5 \text{ amps}$

$$I_a + I_b + \dots I_n = 0$$

**Solution:**  $I_a + I_b + \dots + I_n = 0$

The currents are placed into the equation with the proper signs.

$$\begin{aligned} I_1 + I_2 + I_3 + I_4 &= 0 \\ 10 \text{ amps} + (-3 \text{ amps}) + I_3 + (-5 \text{ amps}) &= 0 \\ I_3 + 2 \text{ amps} &= 0 \\ I_3 &= -2 \text{ amps} \end{aligned}$$

$I_3$  has a value of 2 amperes, and the negative sign shows it to be a current leaving the junction.

**Example:** Using Figure 7 solve for the magnitude and direction of  $I_3$ .

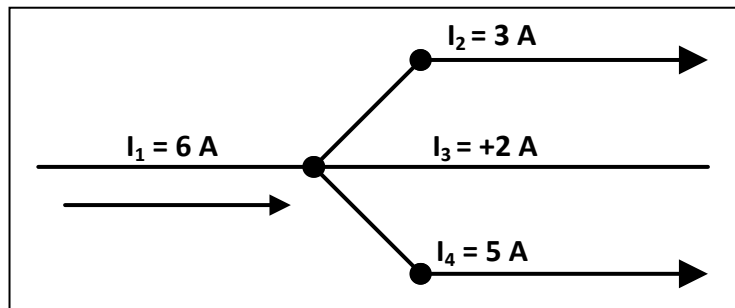


Figure 7: Circuit for example problem

**Given:**  
 $I_1 = 6 \text{ amps}$   
 $I_2 = 3 \text{ amps}$   
 $I_4 = 5 \text{ amps}$

**Solution:**

$$I_a + I_b + \dots I_n = 0$$

$$I_1 + I_2 + I_3 + I_4 = 0$$

$$6 \text{ amps} + (-3 \text{ amps}) + I_3 + (-5 \text{ amps}) = 0$$

$$I_3 + (+2 \text{ amps}) = 0$$

$$I_3 = +2 \text{ amps}$$

$I_3$  is 2 amperes and its positive sign shows it to be a current entering the junction.

### Knowledge Check

1. There is a relationship between total current and current through the individual components in a circuit. What is this relationship in a series circuit and a parallel circuit?
2. In applying Kirchhoff's current law, what does the polarity of the current indicate?
3. What is the relationship of total current to the current through a component in (a) a series circuit, and (b) a parallel circuit?
  - a. (a) divides      (b) divides
  - b. (a) divides      (b) equals
  - c. (a) equals      (b) equals
  - d. (a) equals      (b) divides
4. If a current has a negative polarity when Kirchhoff's current law is applied, which of the following statements is true of the current?
  - a. It is from a battery
  - b. It is from a generator
  - c. It is entering a junction
  - d. It is leaving a junction



## ***Resistance in a Parallel Circuit***

### **Conductance**

When students first see the parallel resistance equation, the natural question to ask is, "Where did *that* thing come from?" It is truly an odd piece of arithmetic, and its origin deserves a good explanation.

Resistance, by definition, is the measure of friction a component presents to the flow of electrons through it. Resistance is symbolized by the capital letter "R" and is measured in the unit of "ohm." However, we can also think of this electrical property in terms of its inverse: how *easy* it is for electrons to flow through a component, rather than how *difficult*.

Resistance is the word we use to measure of how *difficult* it is for electrons to flow.

**Conductance** is the measure of how *easy* it is for electrons to flow.

Mathematically, conductance is the reciprocal, or inverse, of resistance:

$$\text{Conductance} = \frac{1}{\text{Resistance}}$$

**Equation 4: Conductance**

The greater the resistance, the less the conductance, and vice versa. This should make intuitive sense, resistance and conductance being opposite ways to denote the same electrical property. If two components' resistances are compared and it is found that component "A" has one-half the resistance of component "B," then we could alternatively express this relationship by saying that component "A" is twice as conductive as component "B." If component "A" has but one-third the resistance of component "B," then we could say it is three times more conductive than component "B," and so on.

Carrying this idea further, a symbol and unit were created to represent conductance. The symbol is the capital letter "G" and the unit is the **mho**, which is "ohm" spelled backwards.

Despite its appropriateness, the unit of the mho was replaced in later years by the unit of **siemens** (abbreviated by the capital letter "S").

Back to our parallel circuit example, we should be able to see that multiple paths (branches) for current reduces total resistance for the whole circuit, as electrons are able to flow easier through the whole network of multiple branches than through any one of those branch resistances alone. In terms of resistance, additional branches result in a lesser total (current meets with less opposition). In terms of conductance, however, additional branches results in a greater total (electrons flow with greater conductance):

Total parallel resistance is less than any one of the individual branch resistances because parallel resistors resist less together than they would separately:

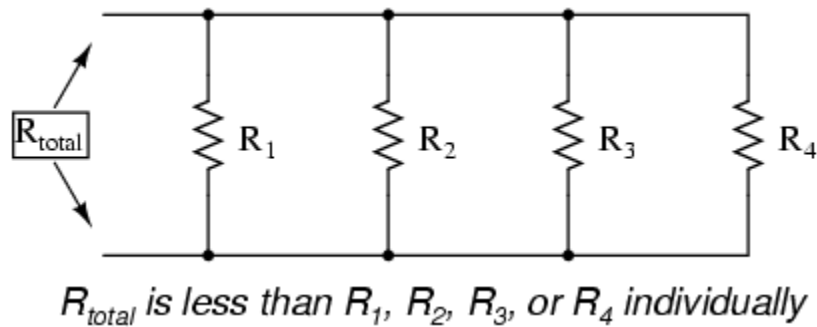
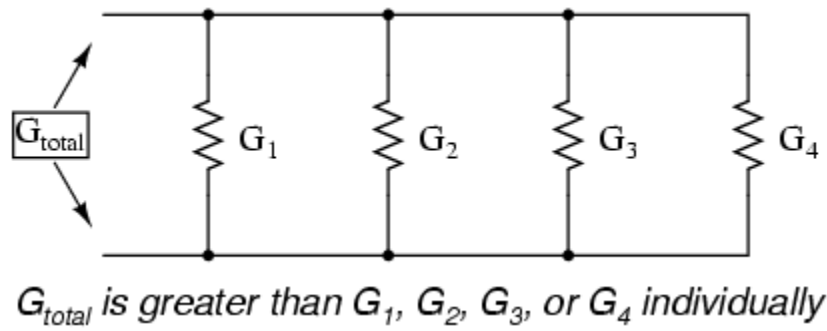


Figure 8: Resistance in Parallel

Total parallel conductance is greater than any of the individual branch conductances because parallel resistors conduct better together than they would separately:



**Figure 9: Conductance in parallel.**

To be more precise, the total conductance in a parallel circuit is equal to the sum of the individual conductances:

$$G_{total} = G_1 + G_2 + G_3 + G_4$$

**Equation 5: Parallel Conductance**

If we know that conductance is nothing more than the mathematical reciprocal ( $1/x$ ) of resistance, we can translate each term of the above formula into resistance by substituting the reciprocal of each respective conductance:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

**Figure 10: Resistance in Parallel**

Solving the above equation for total resistance (instead of the reciprocal of total resistance), we can invert (reciprocate) both sides of the equation:

$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}$$

**Figure 11: Reciprocal Formula**

Conductance (G) is seldom used as a practical measurement, and so the above formula is a common one to see in the analysis of parallel circuits. Below, the various methods for calculating parallel resistance will be discussed including:

- Two equal resistors in parallel
- Two unequal resistors in parallel
- Reciprocal method
- Product over sum method

### Knowledge Check

1. What is the unit of conductance and what other term is sometimes used?
2. What is the relationship between conductance and resistance?

### Two Equal Resistors in Parallel

In the example diagram, Figure 12, there are two resistors connected in parallel across a 5-volt battery. Each has a resistance value of 10 ohms. A complete circuit consisting of two parallel paths is formed and current flows as shown.

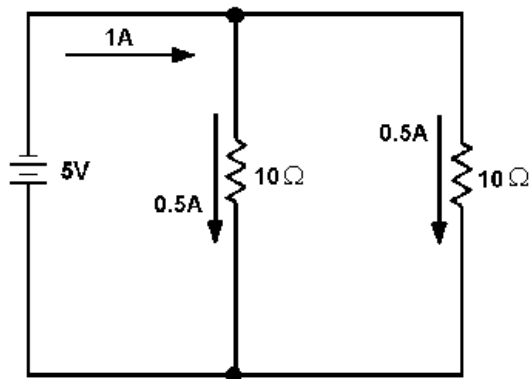


Figure 12: Two equal resistors connected in parallel.

Computing the individual currents shows that there is one-half of an ampere of current through each resistance. The total current flowing from the battery to the junction of the resistors, and returning from the resistors to the battery, is equal to 1 ampere.

The total resistance of the circuit can be calculated by using the values of total voltage ( $E_T$ ) and total current ( $I_T$ ).

**Given:**

$$E_T = 5 \text{ V}$$
$$I_T = 1 \text{ A}$$

**Solution:**

$$R = \frac{E}{I}$$
$$R_T = \frac{E_T}{I_T}$$
$$R_T = \frac{5 \text{ V}}{1 \text{ A}}$$
$$R_T = 5 \Omega$$

This computation shows the total resistance to be 5 ohms; one-half the value of either of the two resistors.

Since the total resistance of a parallel circuit is smaller than any of the individual resistors, **total resistance** of a parallel circuit is not the sum of the individual resistor values as was the case in a series circuit. The total resistance of resistors in parallel is also referred to as **equivalent resistance** ( $R_{eq}$ ). The terms total resistance and equivalent resistance are used interchangeably.

There are several methods used to determine the equivalent resistance of parallel circuits. The best method for a given circuit depends on the number and value of the resistors. For the circuit described above, where all resistors have the same value, the following simple equation is used:

$$R_{eq} = \frac{R}{N}$$

$R_{eq}$  = equivalent parallel resistance

$R$  = ohmic value of one resistor

$N$  = number of resistors

This equation is valid for any number of parallel resistors of equal value.

**Example:** Four 40-ohm resistors are connected in parallel. What is their equivalent resistance?

**Given:**  $R_1 + R_2 + R_3 + R_4$   
 $R_1 = 40 \Omega$

**Solution:**  $R_{eq} = \frac{R}{N}$   
 $R_{eq} = \frac{40\Omega}{4}$   
 $R_{eq} = 10 \Omega$

### Two Unequal Resistors in Parallel

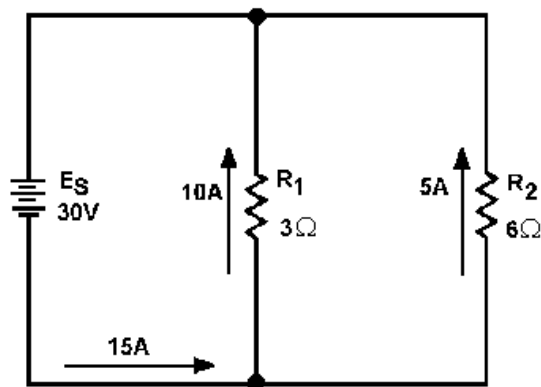


Figure 13: Example circuit with unequal parallel

Figure 13 shows two resistors of unequal value in parallel. Since the total current is shown, the equivalent resistance can be calculated.

**Given:**  $E_s = 30 \text{ V}$   
 $I_T = 15 \text{ A}$

**Solution:**  $R_{eq} = \frac{E_s}{I_T}$   
 $R_{eq} = \frac{30 \text{ V}}{15 \text{ A}}$   
 $R_{eq} = 2 \Omega$

The equivalent resistance of the circuit shown in Figure 13 is smaller than either of the two resistors ( $R_1, R_2$ ). An important point to remember is that **the equivalent resistance of a parallel circuit is always less than the resistance of any branch.**

Equivalent resistance can be found if you know the individual resistance values and the source voltage. By calculating each branch current, adding the branch currents to calculate total current, and dividing the source voltage by the total current, the total can be found. This method, while effective, is somewhat lengthy. A quicker method of finding equivalent resistance is to use the general formula for resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$

**Equation 6: General Formula for Parallel Resistance**

If you apply the general formula to the circuit shown in Figure 13 you will get the same value for equivalent resistance ( $2\Omega$ ) as was obtained in the previous calculation that used source voltage and total current.

**Given:**  $R_1 = 3\Omega$   
 $R_2 = 6\Omega$

**Solution:**

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{3\Omega} + \frac{1}{6\Omega}$$

Convert the fractions to a common denominator.

$$\frac{1}{R_{eq}} = \frac{2}{6\Omega} + \frac{1}{6\Omega}$$

$$\frac{1}{R_{eq}} = \frac{3}{6\Omega}$$

$$\frac{1}{R_{eq}} = \frac{1}{2\Omega}$$

Since both sides are reciprocals (divided into one), disregard the reciprocal function.

$$R_{eq} = 2\Omega$$

The formula you were given for equal resistors in parallel:

$$( R_{eq} = \frac{R}{N} )$$

is a simplification of the general formula for resistors in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$

There are other simplifications of the general formula for resistors in parallel which can be used to calculate the total or equivalent resistance in a parallel circuit.



## Reciprocal Method

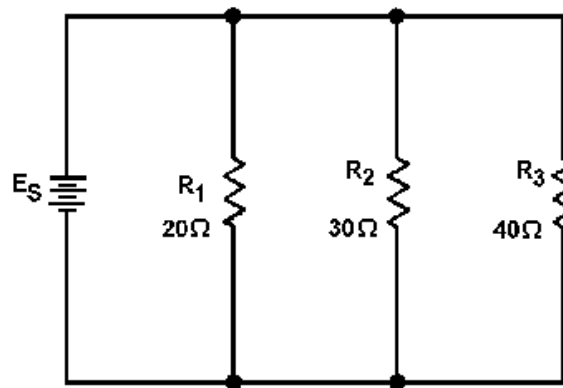
This method is based upon taking the reciprocal of each side of the equation. This presents the general formula for resistors in parallel as:

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}$$

**Equation 7: Reciprocal Method**

This formula is used to solve for the equivalent resistance of a number of unequal parallel resistors. You must find the lowest common denominator in solving these problems

**Example:** Three resistors are connected in parallel as shown in Figure 14. The resistor values are:  $R_1 = 20 \text{ ohms}$ ,  $R_2 = 30 \text{ ohms}$ ,  $R_3 = 40 \text{ ohms}$ . What is the equivalent resistance? (Use the reciprocal method.)



**Figure 14: Example parallel circuit with unequal branch resistors.**

**Given:**

- $R_1 = 20\Omega$
- $R_2 = 30\Omega$
- $R_3 = 40\Omega$

**Solution:**

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$R_{eq} = \frac{1}{\frac{1}{20\Omega} + \frac{1}{30\Omega} + \frac{1}{40\Omega}}$$

$$R_{eq} = \frac{1}{\frac{6}{120\Omega} + \frac{4}{120\Omega} + \frac{3}{120\Omega}}$$

$$R_{eq} = \frac{1}{\frac{13}{120}\Omega}$$

$$R_{eq} = \frac{120}{13}\Omega$$

$$R_{eq} = 9.23\Omega$$

### Calculator Hint

When using a calculator to find the equivalent resistor, use the inverse button ( $1/x$  or  $1^\sim$ ). Using the  $1^\sim$  button and using parenthesis the solution for the example above might look like this:

$$(20^{-1} + 30^{-1} + 40^{-1})^{-1} = 9.23\Omega$$

### Product over Sum Method

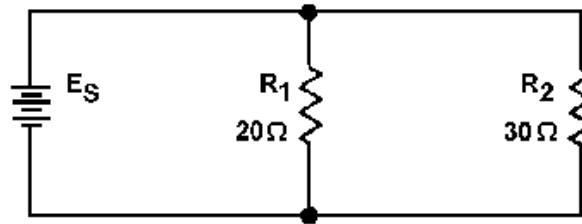
A convenient method for finding the equivalent, or total, resistance of two parallel resistors is by using the following formula.

$$R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2}$$

**Equation 8: Product over Sum Method**

This equation, called the product over the sum formula, is used so frequently it should be committed to memory.

**Example:** What is the equivalent resistance of a 20-ohm and a 30-ohm resistor connected in parallel, as in Figure 15?



**Figure 15: Parallel circuit with two unequal resistors.**

**Given:**  $R_1 = 20\Omega$   
 $R_2 = 30\Omega$

**Solution:**

$$R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2}$$

$$R_{eq} = \frac{20\Omega \times 30\Omega}{20\Omega + 30\Omega}$$

$$R_{eq} = \frac{600}{50} \Omega$$

$$R_{eq} = 12\Omega$$

### Knowledge Check

1. Four equal resistors are connected in parallel; each resistor has an ohmic value of 100 ohms, what is the equivalent resistance?
2. Three resistors connected in parallel have values of 12 k $\Omega$ , 20 k $\Omega$ , and 30 k $\Omega$ . What is the equivalent resistance?
3. Two resistors connected in parallel have values of 10 k $\Omega$  and 30 k $\Omega$ . What is the equivalent resistance?

## ***Embedded Media***

### **Internet Archive - FEDFLIX**

- Electricity – Parallel Resistive Circuits - Analysis
  - <https://archive.org/details/gov.dod.dimoc.39875>
    - The basic theory remains the same even though the technology seems dated.

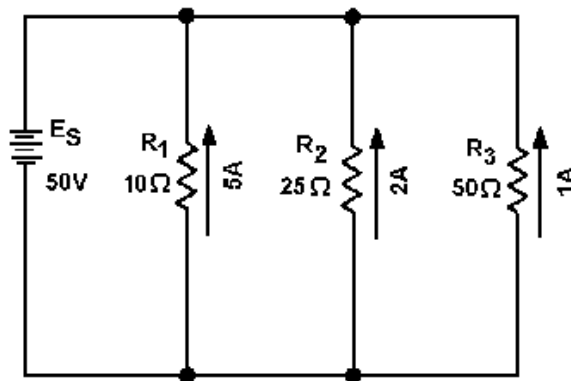
## **Power in a Parallel Circuit**

Power computations in a parallel circuit are the same as those used for the series circuit. Since power dissipation in resistors consists of a heat loss, power dissipations are additive regardless of how the resistors are connected in the circuit. The total power is equal to the sum of the power dissipated by the individual resistors. Like the series circuit, the total power consumed by the parallel circuit is:

$$P_T = P_1 + P_2 + \dots P_n$$

**Equation 9: Power in parallel.**

**Example:** Find the total power consumed by the circuit in Figure 16.



**Figure 16: Example parallel circuit.**

**Given:**  $R_1 = 10 \Omega$   
 $I_{R1} = 5 \text{ A}$

$R_2 = 25 \Omega$   
 $I_{R2} = 2 \text{ A}$

$R_3 = 50 \Omega$   
 $I_{R3} = 1 \text{ A}$

**Solution:**  $P = I^2 R$   
 $P_{R1} = (I_{R1})^2 \times R_1$   
 $P_{R1} = (5 \text{ A})^2 \times 10 \Omega$   
 $P_{R1} = 250 \text{ W}$   
 $P_{R2} = (I_{R2})^2 \times R_2$   
 $P_{R2} = (2 \text{ A})^2 \times 25 \Omega$   
 $P_{R2} = 100 \text{ W}$   
 $P_{R3} = (I_{R3})^2 \times R_3$   
 $P_{R3} = (1 \text{ A})^2 \times 50 \Omega$   
 $P_{R3} = 50 \text{ W}$   
 $P_T = P_{R1} + P_{R2} + P_{R3}$   
 $P_T = 250 \text{ W} + 100 \text{ W} + 50 \text{ W}$   
 $P_T = 400 \text{ W}$

Since the total current and source voltage are known, the total power can also be computed by:

**Given:**  $E_s = 50 \text{ V}$   
 $I_T = 8 \text{ A}$

**Solution:**  $P_T = E_s \times I_T$   
 $P_T = 50 \text{ V} \times 8 \text{ A}$   
 $P_T = 400 \text{ W}$

## ***Knowledge Check***

1. The total power in both series and parallel circuits is computed with the formula:  $P_T = P_1 + P_2 + P_3 + \dots P_n$ . Why can this formula be used for both series and parallel circuits?

## **Rules for Parallel DC Circuits**

- The same voltage exists across each branch of a parallel circuit and is equal to the source voltage.
- The current through a branch of a parallel network is inversely proportional to the amount of resistance of the branch.
- The total current of a parallel circuit is equal to the sum of the individual branch currents of the circuit.
- The total resistance of a parallel circuit is found by the general formula:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots \frac{1}{R_n}$$

Or one of the formulas derived from this general formula.

- The total power consumed in a parallel circuit is equal to the sum of the power consumptions of the individual resistances.

## **Solving Parallel Circuit Problems**

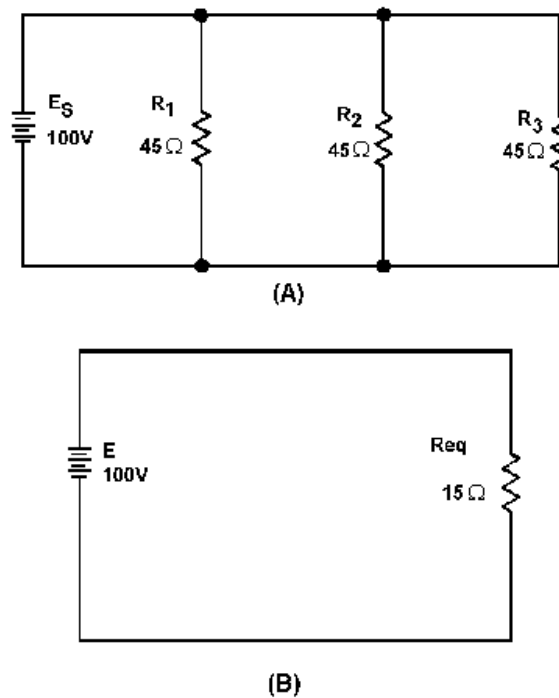
Problems involving the determination of resistance, voltage, current, and power in a parallel circuit are solved as simply as in a series circuit. The procedure is the same:

1. Draw the circuit diagram.
2. State the values given and the values to be found.
3. Select the equations to be used in solving for the unknown quantities based upon the known quantities.
4. Substitute the known values in the equation you have selected and solve for the unknown value.

### ***Equivalent Circuits***

In the study of electricity, it is often necessary to reduce a complex circuit into a simpler form. Any complex circuit consisting of resistances can be redrawn (reduced) to a basic equivalent circuit containing the voltage source and a single resistor representing total resistance. This process is called reduction to an equivalent circuit.

Figure 17 shows a parallel circuit with three resistors of equal value and the redrawn equivalent circuit. The parallel circuit shown in part A shows the original circuit.



**Figure 17: Parallel circuit with equivalent circuit.**

To create the equivalent circuit, you must first calculate the equivalent resistance.

**Given:**

$$R_1 = 45\Omega$$

$$R_2 = 45\Omega$$

$$R_3 = 45\Omega$$

**Solution:**

$$R_{eq} = \frac{R}{N}$$

$$R_{eq} = \frac{45\Omega}{3}$$

$$R_{eq} = 15\Omega$$

Once the equivalent resistance is known, a new circuit is drawn consisting of a single resistor (to represent the equivalent resistance) and the voltage source, as shown in part B.

**Example:** A parallel circuit consists of five resistors. The value of each resistor is known and the current through  $R_1$  is known. You are asked to calculate the value for total resistance, total power, total current, source voltage, the power used by each resistor, and the current through resistors  $R_2$ ,  $R_3$ ,  $R_4$ , and  $R_5$ .

**Given:**

$$R_1 = 20\Omega$$

$$R_2 = 30\Omega$$

$$R_3 = 18\Omega$$

$$R_4 = 18\Omega$$

$$R_5 = 18\Omega$$

$$I_{R1} = 9A$$

**Find:**

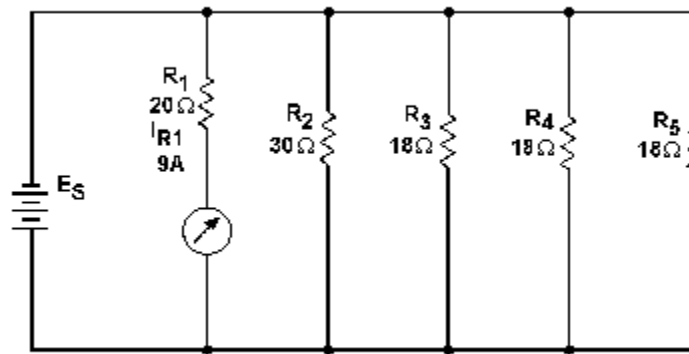
$$R_T, E_s, I_T, P_T, I_{R2}, I_{R3}, I_{R4},$$

$$I_{R5}, P_{R1}, P_{R2}, P_{R3}, P_{R4}, P_{R5}$$

**Solution:** This may appear to be a large amount of mathematical manipulation. However, if you use the step-by-step approach, the circuit will fall apart quite easily.

The first step in solving this problem is for you to draw the circuit and indicate the known values as shown in Figure 18.





**Figure 18: Parallel circuit problem.**

There are several ways to approach this problem. With the values you have been given, you could first solve for  $R_T$ , the power used by  $R_1$ , or the voltage across  $R_1$ , which you know is equal to the source voltage and the voltage across each of the other resistors. Solving for  $R_T$  or the power used by  $R_1$  will not help in solving for the other unknown values.

Once the voltage across  $R_1$  is known, this value will help you calculate other unknowns. Therefore the logical unknown to solve for is the source voltage (the voltage across  $R_1$ ).

**Given:**

$$R_1 = 20\Omega$$

$$I_{R1} = 9A$$

$$E_{R1} = E_S$$

**Solution:**

$$E_S = R_1 \times I_{R1}$$

$$E_S = 9A \times 20\Omega$$

$$E_S = 180V$$

Now that source voltage is known, you can solve for current in each branch.

**Given:**

$$E_s = 180V$$

$$R_2 = 30\Omega$$

$$R_3 = 18\Omega$$

$$R_4 = 18\Omega$$

$$R_5 = 18\Omega$$

**Solution:**  $I_{R2} = \frac{E_s}{R_2}$

$$I_{R2} = \frac{180 V}{30 \Omega}$$

$$I_{R2} = 6 A$$

$$I_{R3} = \frac{E_s}{R_3}$$

$$I_{R3} = \frac{180 V}{18 \Omega}$$

$$I_{R3} = 10 A$$

Since  $R_3 = R_4 = R_5$  and the voltage across each branch is the same:

$$I_{R4} = 10A$$

$$I_{R5} = 10A$$

Solving for total resistance.

**Given:**  $R_1 = 20\Omega$

$$R_2 = 30\Omega$$

$$R_3 = 18\Omega$$

$$R_4 = 18\Omega$$

$$R_5 = 18\Omega$$

**Solution:**  $R_T = R_{eq}$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}$$

$$\frac{1}{R_T} = \frac{1}{20\Omega} + \frac{1}{30\Omega} + \frac{1}{18\Omega} + \frac{1}{18\Omega} + \frac{1}{18\Omega}$$

$$\frac{1}{R_T} = \frac{9 + 6 + 10 + 10 + 10\Omega}{180 \text{ (LCD)}}$$

$$R_T = \frac{45\Omega}{180}$$

$$R_T = \frac{180}{45\Omega}$$

$$R_T = 4\Omega$$

An alternate method for solving for  $R_T$  can be used. By observation, you can see that  $R_3$ ,  $R_4$ , and  $R_5$  are of equal ohmic value. Therefore an equivalent resistor can be substituted for these three resistors in solving for total resistance.

**Given:**  $R_3 = R_4 = R_5 = 18\Omega$

**Solution:**  $R_{eq1} = \frac{R}{N}$

$$R_{eq1} = \frac{18\Omega}{3}$$

$$R_{eq1} = 6\Omega$$

The circuit can now be redrawn using a resistor labeled  $R_{eq1}$  in place of  $R_3$ ,  $R_4$ , and  $R_5$  as shown in Figure 19.

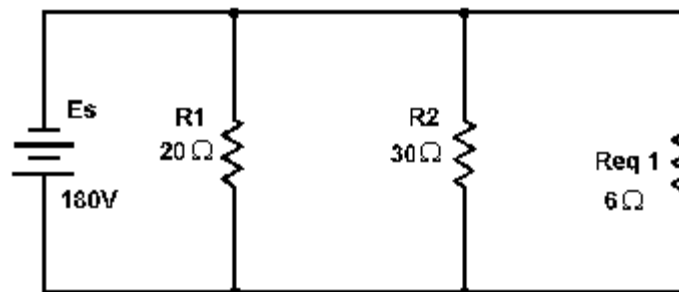


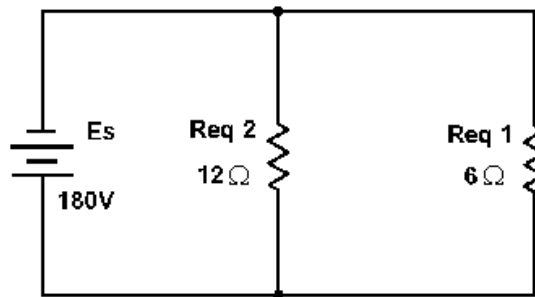
Figure 19: First equivalent parallel circuit.

An equivalent resistor can be calculated and substituted for  $R_1$  and  $R_2$  by use of the **product over the sum formula**.

**Given:**  $R_1 = 20\Omega$   
 $R_2 = 30\Omega$

**Solution:**  $R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2}$   
 $R_{eq2} = \frac{20\Omega \times 30\Omega}{20\Omega + 30\Omega}$   
 $R_{eq2} = \frac{600}{50}\Omega$   
 $R_{eq2} = 12\Omega$

The circuit is now redrawn again using a resistor labeled  $R_{eq2}$  in place of  $R_1$  and  $R_2$  as shown in Figure 20.



**Figure 20: Second equivalent parallel circuit.**

You are now left with two resistors in parallel. The product over the sum method can now be used to solve for total resistance.

**Given:**  $R_{eq1} = 6\Omega$   
 $R_{eq2} = 12\Omega$   
 $R_T = R_{eq}$

**Solution:**

$$R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2}$$

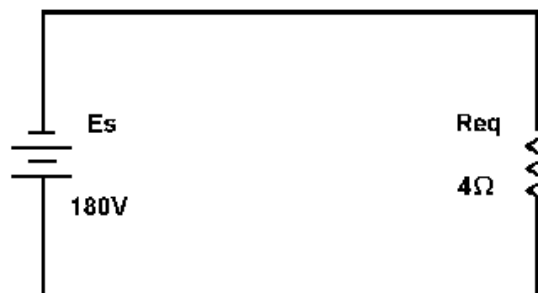
$$R_T = \frac{R_{eq1} \times R_{eq2}}{R_{eq1} + R_{eq2}}$$

$$R_T = \frac{6\Omega \times 12\Omega}{6\Omega + 12\Omega}$$

$$R_T = \frac{72}{18} \Omega$$

$$R_T = 4\Omega$$

This agrees with the solution found by using the general formula for solving for resistors in parallel. The circuit can now be redrawn as shown in Figure 21 and total current can be calculated.



**Figure 21: Parallel circuit redrawn to final equivalent circuit.**

**Given:**

$$E_s = 180V$$

$$R_T = 4\Omega$$

**Solution:**

$$I_T = \frac{E_s}{R_T}$$

$$I_T = \frac{180V}{4\Omega}$$

$$I_T = 45A$$

This solution can be checked by using the values already calculated for the branch currents.

**Given:**

$$I_{R1} = 9A$$
$$I_{R2} = 6A$$
$$I_{R3} = 10A$$
$$I_{R4} = 10A$$
$$I_{R5} = 10A$$

**Solution:**

$$I_T = I_{R1} + I_{R2} + \dots I_{Rn}$$
$$I_T = 9A + 6A + 10A + 10A + 10A$$
$$I_T = 45A$$

Now that total current is known, the next logical step is to find total power.

**Given:**

$$E_s = 180V$$
$$I_T = 45A$$

**Solution:**

$$P = EI$$
$$P_T = E_s \times I_T$$
$$P_T = 180V \times 45A$$
$$P_T = 8100 \text{ watts} = 8.1 \text{ kW}$$

Solving for the power in each branch.

**Given:**

$$E_s = 180V$$
$$I_{R1} = 9A$$
$$I_{R2} = 6A$$
$$I_{R3} = 10A$$
$$I_{R4} = 10A$$
$$I_{R5} = 10A$$

**Solution:**

$$P = EI$$

$$P_{R1} = E_s \times I_{R1}$$

$$P_{R1} = 180V \times 9A$$

$$P_{R1} = 1620W$$

$$P_{R2} = E_s \times I_{R2}$$

$$P_{R2} = 180V \times 6A$$

$$P_{R2} = 1080W$$

$$P_{R3} = E_s \times I_{R3}$$

$$P_{R3} = 180V \times 10A$$

$$P_{R3} = 1800W$$

Since  $I_{R3} = I_{R4} = I_{R5}$  then,  $P_{R3} = P_{R4} = P_{R5} = 1800 W$ . The previous calculation for total power can now be checked.

**Given:**

$$P_{R1} = 1620W$$

$$P_{R2} = 1080W$$

$$P_{R3} = 1800W$$

$$P_{R4} = 1800W$$

$$P_{R5} = 1800W$$

**Solution:**

$$P_T = P_{R1} + P_{R2} + P_{R3} + P_{R4} + P_{R5}$$

$$P_T = 1620W + 1080W + 1800W + 1800W + 1800W$$

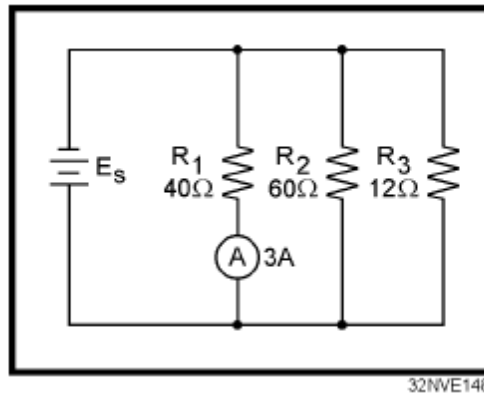
$$P_T = 8100W$$

$$P_T = 8.1kW$$

### ***Knowledge Check***

1. A circuit consists of three resistors connected in parallel.  $R_1 = 30$  ohms,  $R_2 = 15$  ohms, and  $R_3 = 10$  ohms. If the current through  $R_2 = 4$  amperes, what is the total source voltage?
  - a. 20 V
  - b. 60 V
  - c. 120 V
  - d. 220 V

2. What term identifies a single resistor that represents total resistance of a complex circuit?
3. A circuit consists of three resistors connected in parallel across a voltage source.  $R_1 = 40\Omega$ ,  $R_2 = 30\Omega$ ,  $R_3 = 40\Omega$ , and  $P_{R3} = 360$  watts. Solve for  $R_T$ ,  $E_S$  and  $I_{R2}$ . (Hint: Draw and label the circuit first.)



32NVE148

Questions 4 – 8 refer to the circuit above.

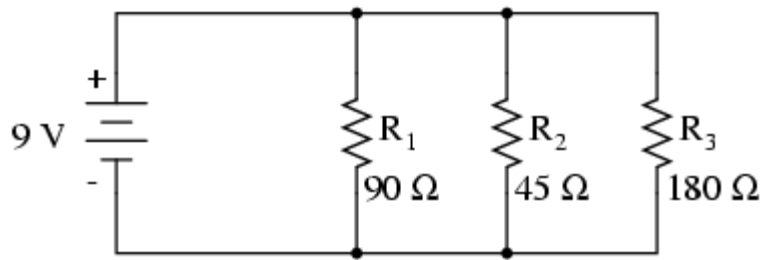
4. What is the value of  $E_S$ ?
  - a. 336 V
  - b. 300 V
  - c. 240 V
  - d. 120 V
5. What is the value of current through  $R_2$ ?
  - a. 1 A
  - b. 2 A
  - c. 3 A
  - d. 4 A
6. What is the approximate value of total resistance?
  - a.  $8\Omega$
  - b.  $37\Omega$
  - c.  $112\Omega$
  - d.  $257\Omega$



7. What is the value of total power?
- e. 1.2 kW
  - f. 1.5 kW
  - g. 1.8 kW
  - h. 2.0 kW
8. What is the total power consumed by  $R_3$ ?
- i. 108 W
  - j. 240 W
  - k. 360 W
  - l. 1200 W

## Troubleshooting Parallel Circuits

As stated before, the job of a technician requires good troubleshooting skills. The same analysis technique as shown for series circuits works for parallel circuits as well. First, we determine what a "healthy" parallel circuit should behave like and record those values in a table as shown below.



	$R_1$	$R_2$	$R_3$	Total	
E	9	9	9	9	Volts
I	100m	200m	50m	350m	Amps
R	90	45	180	25.714	Ohms

Figure 22: A healthy parallel circuit.

Supposing that  $R_2$  opens in this parallel circuit, here's what the effects will be:

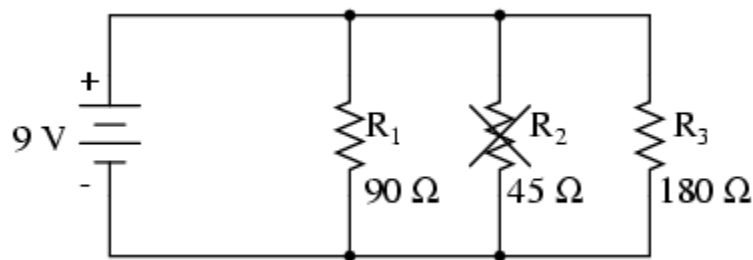


Figure 23:  $R_2$  becomes open.

Notice in Figure 24 that an open branch only affects the current through that branch and the circuit's total current. Total voltage -- being shared equally across all components in a parallel circuit, will be the same for all resistors. Due to the fact that the voltage source's tendency is to hold voltage *constant*, its voltage will not change, and being in parallel with all the resistors, it will hold all the resistors' voltages the same as they were before: 9 volts. Being that voltage is the only common parameter in a parallel circuit, and the other resistors haven't changed resistance value, their respective branch currents remain unchanged.

	$R_1$	$R_2$	$R_3$	Total	
E	9	9	9	9	Volts
I	100m	0	50m	150m	Amps
R	90	$\infty$	180	60	Ohms


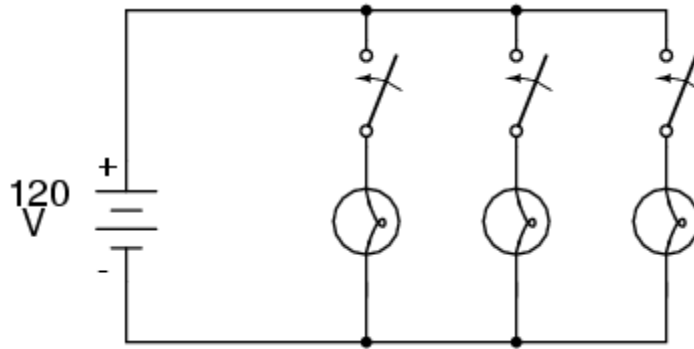

  
*Open resistor*

Figure 24: Effects of  $R_2$  opened.

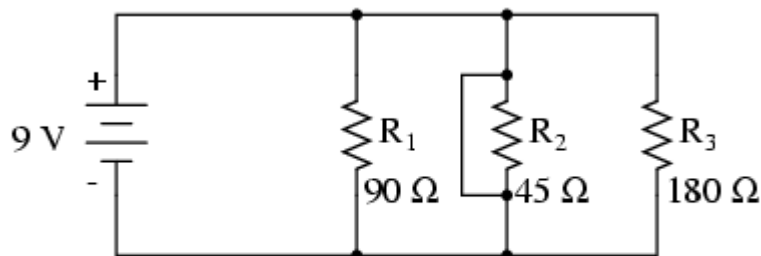
This is what happens in a household lamp circuit (Figure 25): all lamps get their operating voltage from power wiring arranged in a parallel fashion. Turning one lamp on and off (one branch in that parallel circuit closing and opening) doesn't affect the operation of other lamps

in the room, only the current in that one lamp (branch circuit) and the total current powering all the lamps in the room:



**Figure 25: Household lamp circuit.**

In an ideal case (with perfect voltage sources and zero-resistance connecting wire), shorted resistors in a simple parallel circuit will also have no effect on what's happening in other branches of the circuit. In real life, the effect is not quite the same, and we'll see why in the following circuit:



*$R_2$  "shorted" with a jumper wire*

**Figure 26: R2 shorted**

A shorted resistor (resistance of  $0\ \Omega$ ) would theoretically draw infinite current from any finite source of voltage ( $I=E/0$ ). In this case, the zero resistance of  $R_2$  decreases the circuit total resistance to zero  $\Omega$  as well, increasing total current to a value of infinity. As long as the voltage source holds steady at 9 volts, however, the other branch currents ( $I_{R1}$  and  $I_{R3}$ ) will remain unchanged (Figure 27).

	$R_1$	$R_2$	$R_3$	Total	
E	9	9	9	9	Volts
I	100m	$\infty$	50m	$\infty$	Amps
R	90	0	180	0	Ohms



  
*Shorted resistor*

Figure 27: Effects of R2 shorted.

To determine what would happen in a circuit if a component fails, re-draw that circuit with the equivalent resistance of the failed component in place, and re-calculate all values.

The ability to intuitively determine what will happen to a circuit with any given component fault is a crucial skill for any electronics troubleshooter to develop. The best way to learn is to experiment with circuit calculations and real-life circuits, paying close attention to what changes with a fault, what remains the same, and why!

### Reminders

- A **shorted** component is one whose resistance has dramatically decreased.
- An **open** component is one whose resistance has dramatically increased. For the record, resistors tend to fail open more often than fail shorted, and they almost never fail unless physically or electrically overstressed (physically abused or overheated).

### Embedded Media

#### Internet Archive - FEDFLIX

- Electricity – Parallel Resistive Circuits – Power-Troubleshooting
  - <https://archive.org/details/gov.dod.dimoc.39876>
    - January, 1975. The basic theory remains the same even though the technology seems dated.

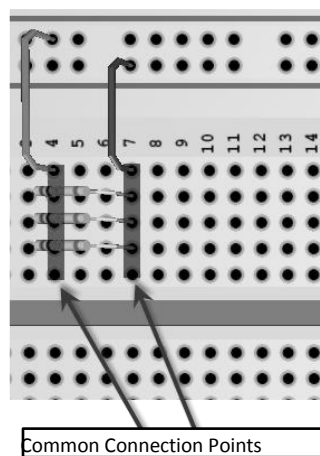
## Parallel Circuits Lab 1: Simple Parallel Circuit

### *How to wire a breadboard in Parallel*

Up to now, all of the breadboard circuits you have created have been simple series circuits. Parallel circuits on the breadboard are a bit more difficult and require attention to detail. The easiest way to start is to build the circuit to look as much like the schematic as possible. While this seems simplistic, there will be a point where it becomes intuitive.

In the parallel circuits we will be working with, the main thing to keep in mind is that all parallel branches share common connections with the both ends of the power supply. Translating what you see in the schematic to a breadboard is rather straight forward when you keep that in mind.

Shown in the Figure 28 is a simple parallel circuit with three resistors. The resistors all share a common connection point with both the “hot” (top bus) and the “ground” (bottom bus). It looks very much like the schematic for part one of this lab assignment.

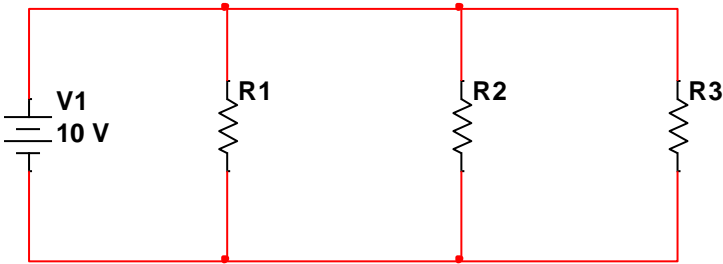


**Figure 28: Breadboard Example**

## ***Components & Equipment Needed***

Resistors, wire, and breadboard

## ***Circuit Diagram***



### **Specifications for Circuit**

- $V_T = 10\text{ V}$
- $R_1 = 4.3\text{ k}\Omega$
- $R_{EQ} = 1\text{ k}\Omega$
- $I_T = 10\text{ mA}$

**Circuit 1: Parallel Circuits Lab 1**

## ***Procedure***

**Step 1:**      **Build the circuit per specifications**

- Build the circuit shown in the schematic using the specifications below in both Multisim and on a breadboard.

**Step 2:**      **Take and record the measurements as shown in Table 1.**

***Table for Parallel Circuits Lab 1***

	Multisim	Breadboard
$V_{R1}$ ( $R_1 = 4.3 \text{ k}\Omega$ )		
$V_{R2}$ ( $R_2 = \underline{\hspace{1cm}}$ )		
$V_{R3}$ ( $R_3 = \underline{\hspace{1cm}}$ )		
$I_T$		
$I_1$		
$I_2$		
$I_3$		
$R_T$		
$R_1$		
$R_2$		
$R_3$		

**Table 1: Parallel Circuit Lab 1*****Observations and Conclusions***

In your lab report, include your results from Table 1 as well as any observations or conclusions you may have made during this exercise.

Include the following in your lab report:

Calculate the values of current and resistance given the fault shown in the chart.

Explain how an open resistor affects total current and resistance.

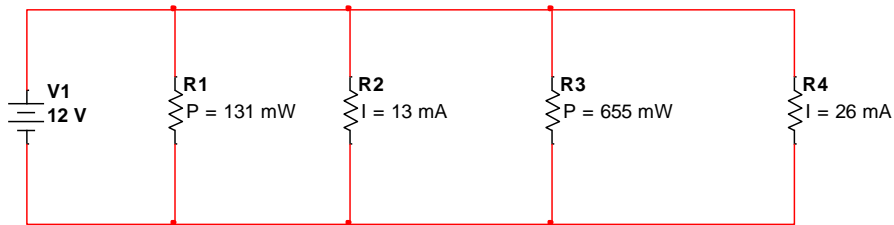
Fault	$R_T$	$I_T$
R1 Open		
R2 Open		
R3 Open		

## Parallel Circuits Lab 2: Solving a Parallel Circuit and Reducing to its Equivalent Circuit

### *Components & Equipment Needed*

- Resistors, wire, and breadboard

### *Circuit Diagram*



Total Power = 1.25 W

**Circuit 2: Parallel Circuits Lab 2**

### *Procedure*

#### **Step 1: Build the circuit per specifications.**

Build the circuit in **both** Multisim **and** on a breadboard as shown in the diagram above using the specification shown in the schematic. Only standard value  $\pm 5\%$  resistors will be used. HINT: If you get stuck, review the information that you have, including the values given and any formulas, laws, or theories you have learned.



**Step 2:**        **Take and record the measurements as shown in Table 2**

***Tables for Parallel Circuits Lab 2***

	Multisim	Breadboard
$V_{R1}$		
$V_{R2}$		
$V_{R3}$		
$V_{R4}$		
$V_T$		
$I_{R1}$		
$I_{R2}$		
$I_{R3}$		
$I_{R4}$		
$I_T$		
$R_1$		
$R_2$		
$R_3$		
$R_4$		
$R_T$		
$P_{R1}$		
$P_{R2}$		
$P_{R3}$		
$P_{R4}$		
$P_T$		

**Table 2: Parallel Circuits Lab 2**

***Observations and Conclusions***

In your lab report, include your results from Table 2 as well as any observations or conclusions you may have made during this exercise.

## Parallel Circuits Lab 3: Critical Thinking

### *The Problem*

Once again, your supervisor approaches you with a problem to solve. Another potential client has asked for a similar circuit as you had designed before. However, they want to be able to operate several different types of bulbs from the same circuit. Now that you have an understanding of parallel circuits, you know that you can solve this problem easily.

Because of industry standards, the total power dissipated must still be 45 W. The client wants three (3) bulbs and the choice of a 12 V or 24 V power supply. The three bulbs desired have resistances of  $10\Omega$ ,  $10\Omega$ , and  $15\Omega$ . If all works well and everything in the circuit is within tolerance ( $\pm 5\%$ ), there won't be need for additional resistance. But if not, there is just enough room for a single standard ( $\pm 5\%$ ) value resistor.

### **Some points to keep in mind:**

- $P_T = 45 \text{ W}$
- Power supply = 12 V or 24 V
- Bulb 1 =  $10 \Omega$
- Bulb 2 =  $10 \Omega$
- Bulb 3 =  $15 \Omega$
- Must consist of only single load (resistance) parallel branches (no series or series-parallel loads)

### **Support all answers with calculations.**

1. Which power supply would be acceptable to use in this case?
2. What is the equivalent resistance of the circuit? Is it within tolerances?

### Problem Solving Help

Before you panic, remember to review the information, or data, that you have for solving this problem. First, you know the total values that are required:

- $V_T = 10 \text{ V}$
- $R_1 = 4.3 \text{ } \Omega$
- $R_{EQ} = 1 \text{ k}\Omega$
- $I_T = 10 \text{ mA}$

The other pieces of information that you have are the formulas, laws, and theories that you have already learned.

- $V_{\text{applied}}$  is the same across parallel branches
- Ohm's Law: For each branch,  $I = V_{\text{applied}}/R$
- Kirchhoff's Current Law: The total current,  $I_T$ , in the main line of a parallel circuit equals the sum of the currents in the individual branches
- Resistors in Parallel follow different rules than those in series.
  - $R_{EQ}$  is less than any branch resistance
  - $R_{EQ} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}$ , reciprocal resistance formula when multiple parallel branches

### Steps to Solving

1. A first step is to review the total, or equivalent, resistance required for the circuit, and any other relevant values.
2. Next, given that the total, or equivalent, resistance will be less than any single branch resistances, choose any reasonable number over  $1 \text{ k}\Omega$  for the value of  $R_2$ .
3. Use a derivative of the reciprocal resistance formula to determine the value of  $R_3$ .

## Answers to Knowledge Checks

### *Voltage in a Parallel Circuit*

1. What would the source voltage ( $E_s$ ) in Figure 3 be if the current through  $R_2$  were 2 milliamps?

**60 V**

### *Current in a Parallel Circuit*

1. There is a relationship between total current and current through the individual components in a circuit. What is this relationship in a series circuit and a parallel circuit?

**Total current in a series circuit flows through every circuit component but in a parallel circuit total current divides among the available paths.**

2. In applying Kirchhoff's current law, what does the polarity of the current indicate?

**Whether the current is entering the node (+) or leaving the node (-).**

3. What is the relationship of total current to the current through a component in (a) a series circuit, and (b) a parallel circuit?

- |                      |                    |
|----------------------|--------------------|
| a. (a) divides       | (b) divides        |
| b. (a) divides       | (b) equals         |
| c. (a) equals        | (b) equals         |
| <b>d. (a) equals</b> | <b>(b) divides</b> |

4. If a current has a negative polarity when Kirchhoff's current law is applied, which of the following, statements is true of the current?

- |                                    |
|------------------------------------|
| a. It is from a battery            |
| b. It is from a generator          |
| c. It is entering a junction       |
| <b>d. It is leaving a junction</b> |

### *Resistance in a Parallel Circuit*

1. Four equal resistors are connected in parallel; each resistor has an ohmic value of 100 ohms, what is the equivalent resistance?

**25  $\Omega$  ( $R_{eq} = R/N$ )**

2. Three resistors connected in parallel have values of 12 kΩ, 20 kΩ, and 30 kΩ. What is the equivalent resistance?

$$6 \text{ k}\Omega \left( R_{eq} = \frac{1}{\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right) + \left(\frac{1}{R_3}\right)} \right)$$

3. Two resistors connected in parallel have values of 10 kΩ and 30 kΩ. What is the equivalent resistance?

$$7.5 \text{ k}\Omega \left( R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2} \right)$$

### ***Conductance***

1. What is the unit of conductance and what other term is sometimes used?

**mho, siemens**

2. What is the relationship between conductance and resistance?

**They are reciprocals of each other**

### ***Power in a Parallel Circuit***

1. The total power in both series and parallel circuits is computed with the formula:  $P_T = P_1 + P_2 + P_3 + \dots P_n$ . Why can this formula be used for both series and parallel circuits?

**In both cases all the power used in the circuit must come from the source.**

### ***Solving Parallel Circuit Problems***

1. A circuit consists of three resistors connected in parallel.  $R_1 = 30$  ohms,  $R_2 = 15$  ohms, and  $R_3 = 10$  ohms. If the current through  $R_2 = 4$  amperes, what is the total source voltage?

- a. **20 V**
- b. 60 V
- c. 120 V
- d. 220 V

2. What term identifies a single resistor that represents total resistance of a complex circuit?

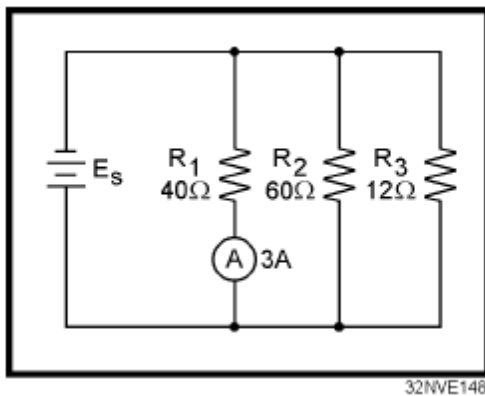
### Equivalent resistance

3. A circuit consists of three resistors connected in parallel across a voltage source.  $R_1 = 40\Omega$ ,  $R_2 = 30\Omega$ ,  $R_3 = 40\Omega$ , and  $P_{R3} = 360$  watts. Solve for  $R_T$ ,  $E_S$  and  $I_{R2}$ . (Hint: Draw and label the circuit first.)

$$R_T = 12\Omega \quad \left( \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$

$$E_S = 120V \quad (E_S = E_{R3} = \sqrt{P_{R3} \times R_3})$$

$$I_{R2} = 4A \quad (I_{R2} = \frac{E_S}{R_2})$$



Questions 4 – 8 refer to the circuit above.

4. What is the value of  $E_S$ ?
  - a. 336 V
  - b. 300 V
  - c. 240 V
  - d. 120 V**
5. What is the value of current through  $R_2$ ?
  - a. 1 A
  - b. 2 A**
  - c. 3 A
  - d. 4 A
6. What is the approximate value of total resistance?
  - a. 8Ω**

- b.  $37\Omega$
  - c.  $112\Omega$
  - d.  $257\Omega$
7. What is the value of total power?
- a. 1.2 kW
  - b. 1.5 kW
  - c. **1.8 kW**
  - d. 2.0 kW
8. What is the total power consumed by R3?
- a. 108 W
  - b. 240 W
  - c. 360 W
  - d. **1200 W**

## **Additional Resources**

### ***Physics Resources***

**Georgia State University – HyperPhysics:** <http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html>

### ***Video Resources***

#### **Khan Academy**

Circuits (part 3): <https://www.khanacademy.org/science/physics/electricity-and-magnetism/v/circuits--part-3>



## References

Kuphaldt, T. (n.d.). *Component Failure Analysis*. Retrieved February 14, 2014, from All About Circuits: [http://www.allaboutcircuits.com/vol\\_1/chpt\\_5/7.html](http://www.allaboutcircuits.com/vol_1/chpt_5/7.html)

Kuphaldt, T. (n.d.). *Conductance*. Retrieved March 19, 2014, from All About Circuits: [http://www.allaboutcircuits.com/vol\\_1/chpt\\_5/4.html](http://www.allaboutcircuits.com/vol_1/chpt_5/4.html)

United States Navy. (2003). Module 1 - Introduction to Matter, Energy, and Direct Current. In *Navy Electricity and Electronics Training Series (NEETS)*. Pensacola, FL: Naval Education and Training Professional Development and Technology Center.

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## Table of Figures

*Source of figures indicated in parentheses. Some figures were modified by Olympic College.*

Figure 1: Example of a basic parallel circuit (US Navy).....	4
Figure 2: Voltage comparison in a parallel circuit (US Navy).....	5
Figure 3: Example problem parallel circuit (US Navy). ....	6
Figure 4: Analysis of current in parallel circuit (US Navy).....	8
Figure 5: Current behavior in parallel circuits (US Navy).....	10
Figure 6: Circuit for example problem (US Navy). ....	13
Figure 7: Circuit for example problem (US Navy) .....	14
Figure 8: Resistance in Parallel (Kuphaldt, T.).....	17
Figure 9: Conductance in parallel (Kuphaldt, T.).....	18
Figure 10: Resistance in Parallel (Kuphaldt, T.).....	18
Figure 11: Reciprocal Formula (Kuphaldt, T.).....	18
Figure 12: Two equal resistors connected in parallel (US Navy)l. ....	19
Figure 13: Example circuit with unequal parallel resistors (US Navy).....	21
Figure 14: Example parallel circuit with unequal branch resistors (US Navy).....	24
Figure 15: Parallel circuit with two unequal resistors (US Navy).....	26
Figure 16: Example parallel circuit (US Navy). ....	27
Figure 17: Parallel circuit with equivalent circuit (US Navy).....	30
Figure 18: Parallel circuit problem (US Navy). ....	32
Figure 19: First equivalent parallel circuit (US Navy).....	34
Figure 20: Second equivalent parallel circuit (US Navy).....	35
Figure 21: Parallel circuit redrawn to final equivalent circuit (US Navy).....	36
Figure 22: A healthy parallel circuit (Kuphaldt, T.).....	40
Figure 23: R2 becomes open (Kuphaldt, T.).....	41
Figure 24: Effects of R2 opened (Kuphaldt, T.). ....	41
Figure 25: Household lamp circuit (Kuphaldt, T.). ....	42
Figure 26: R2 shorted (Kuphaldt, T.) .....	42
Figure 27: Effects of R2 shorted (Kuphaldt, T.). ....	43
Figure 28: Breadboard Example (Olympic College) .....	44

## Table of Tables

*Created by Olympic College*

Table 1: Parallel Circuit Lab 1 .....	46
Table 2: Parallel Circuits Lab 2 .....	48

## Table of Circuits

*Unless otherwise noted, created using National Instruments Multisim v. 12.*

Circuit 1: Parallel Circuits Lab 1 .....	45
Circuit 2: Parallel Circuits Lab 2 .....	47

## Table of Equations

*Created by Olympic College*

Equation 1: Voltage in a parallel circuit. ....	5
Equation 2: Current in a parallel circuit. ....	10
Equation 3: Kirchhoff's Current Law .....	13
Equation 4: Conductance .....	16
Equation 5: Parallel Conductance .....	18
Equation 6: General Formula for Parallel Resistance .....	22
Equation 7: Reciprocal Method .....	24
Equation 8: Product over Sum Method .....	25
Equation 9: Power in parallel.....	27