## AdUlt Learning Academy

## MATH Algebra I



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Edited by Nicole McMeans St. Louis Community College

## Adult Learning Academy Elementary Algebra Workbook

## ALA Elementary Algebra Math Progress

Name: $\qquad$ Date started: $\qquad$

| DATE |  | SCORE |
| :--- | :--- | :--- |
| Module 1: |  |  |
| Variables, Expressions, Equations, and Inequalities |  |  |
| Module 2: <br> Applications of Linear Equations, Function Notation |  |  |
| Module 3: <br> Rectangular Coordinate System, Graphs of Linear Equations, <br> Slope |  |  |
| Module 4: <br> Equations of Lines, Graphs of Linear Inequalities, Solving <br> Systems by Graphing |  |  |
| Module 5: <br> Integer Exponents and Laws, Scientific Notation, Polynomials <br> and Operations |  |  |
| Module 6: <br> Factoring Polynomials, Solving Quadratic Equations by <br> Factoring |  |  |
| Retake Compass Test |  |  |

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Module 1: Expressions, Equations, and Inequalities Learning Objectives
By the time you finish this module, you should be able to:
$\square$ Review topic: Add, subtract, multiply, and divide fractions and integers (neg, pos)
$\square$ Write and simplify algebraic expressions using variables
$\square$ Recognize like terms; add and subtract them appropriately
Distinguish between expressions to be simplified and equations to be solved
Solve linear equations with one variable, including multi-step equations, equations requiring use of the distributive property, equations containing fractions, and equations with variables on both sidesSolve linear inequalities in one variable and graph their solutions on a number line, using brackets, parentheses, the infinity symbol, set-builder notation, and interval notation appropriately. Also solve compound inequalities.

## IMPORTANT INFORMATION FROM MODULE 1:

Birthday song: You must have common denominators... to ADD or SUBTRACT... Birthday song: You must have like terms... to ADD or SUBTRACT...

Pos + Pos $=$ Pos $\quad$ Neg + Neg $=$ NEG (watch this one!) $\quad$ Pos + Neg $=$ it depends!
To subtract a number, ADD its OPPOSITE
Pos x Pos $=$ Pos $\quad$ Neg x Neg $=$ Pos $\quad$ Pos x Neg $=$ Neg $\quad$ Neg x Pos $=$ Neg
Pos $\div$ Pos $=$ pos $\quad \mathrm{Neg} \div \mathrm{Neg}=\mathrm{Pos} \quad \mathrm{Pos} \div \mathrm{Neg}=\mathrm{Neg} \quad \mathrm{Neg} \div$ Pos $=\mathrm{Neg}$
You can SIMPLIFY expressions, but not solve them.
You can SOLVE equations (they have an = sign!). To SOLVE an equation or inequality means to find the value(s) of the variable that makes the equation or inequality true.

Golden Rule of Algebra: Whatever you do to one side of an equation or inequality, you MUST do to the other side!

To undo addition, subtract. To undo subtraction, add.
To undo multiplication, divide. To undo division, multiply.
You can check the solution to any equation by plugging the value for the variable back into the original equation and see if it satisfies the equation.

If an equation has NO solution, it is called a CONTRADICTION.
If ALL REAL NUMBERS are solutions to an equation, it is called an IDENTITY.
Use brackets [ ] or a closed circle for $\geq$ or $\leq$. Use Parentheses ("Don't Touch Me!") or an open circle for < or > .

When you multiply or divide both sides of an inequality by a negative number, you must FLIP the inequality symbol.

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## Module 1 Video \& Exercise List

| Topic | Website | Videos | Exercises |
| :---: | :---: | :---: | :---: |
| Review Fractions, Integers | www.stlcc.edu | Blackboard Review Flashcards: Fractions |  |
|  |  | Blackboard Review Flashcards: Integers |  |
| If you need more review | ons and integers, see the | w videos and exercises. |  |
| Variables and Expressions | www.khanacademy.org | The Beauty of Algebra |  |
|  |  | Why All the Letters in Algebra? | Evaluating Expressions in 1 Var. |
|  |  | What is a variable? | Combining Like Terms |
|  |  | Why aren't we using the mult sign? | Comb. Like Terms w/Distribution |
|  |  | Variables, Expressions, and equations | Writing Expressions |
|  |  | Example: Evaluating an expression |  |
|  |  | Combining Like Terms |  |
|  |  | Comb. Like Terms \& Distributive Prop |  |
|  |  | Combining Like Terms 1 |  |
|  |  | Combining Like Terms 2 |  |
|  |  | Equation Special Cases |  |
| Solving 1-step equations | www.khanacademy.org | Why do the same thing to both sides? | One-step Equation Intuition |
|  |  | Simple equations | One-step Equations |
|  |  | Representing a relationship w/ equation | One-step equations w/ multipli. |
|  |  | One-step equation intuition | Equations w/ Var. on both sides |
|  |  | 1 -step eq. intuition exercise intro | Worksheets: Solving Equations |
|  |  | Solving one-step equations |  |
|  |  | Solving one-step equations 2 |  |
|  |  | One-step Equations |  |
|  |  | Add/Sub the same thing from both sides |  |
|  |  | Intuition why we divide both sides |  |
|  |  |  |  |
|  |  |  |  |


| Topic | Website | Videos | Exercises |
| :---: | :---: | :---: | :---: |
| Solving 2-step equations | www.khanacademy.org | Why we do the same... 2-step equations | Two-step equations |
|  |  | Why we do the same... Multip-step | Multi-step equations w/ distrib. |
|  |  | Two-step equations | Worksheets: Solving Equations |
|  |  | Variables on both sides |  |
|  |  | Ex. 1 Variables on both sides |  |
|  |  | Ex. 2 Variables on both sides |  |
|  |  | Solving Equations w/ Distributive Prop |  |
|  |  | Ex. 1 Distributive Property to Simplify |  |
|  |  | Ex. 3 Distributive Property to Simplify |  |
| Two-Step | http://www.youtube.com | ch?v=KBpNLjiv8pk |  |
| Combining like terms | http://www.youtube.com/ | ch?v=fXD4DjSyoyo |  |
| Variable on each side | http://www.youtube.com/ | ch? $\mathrm{v}=\mathrm{gQdH5} 5 \mathrm{KWrPQ}$ |  |
| Distributive Property | http://www.youtube.com/ | ch?v=XfaWLVLfeJM |  |
| Solving Inequalities | www.khanacademy.org | Inequalities on a Number Line | Inequalities on a Number Line |
|  |  | Inequalities Using Addition and Subtr. | One-Step Inequalities |
|  |  | Inequalities Using Mult and Division | Multi-Step Linear Inequalities |
|  |  | One-Step Inequalities |  |
|  |  | One-Step Inequalities 2 |  |
|  |  | Solving Inequalities |  |
|  |  | Multi-Step Inequalities |  |
|  |  | Multi-Step Inequalities 2 |  |
|  |  | Multi-Step Inequalities 3 |  |
|  |  | Writing and Using Inequalities 3 |  |
|  |  | Compound Inequalities 3 |  |
|  | www.stlcc.edu | Inequalities Powerpoint on Blackboard |  |
| Module 1 Review | www.stlcc.edu | Powerpoint on Blackboard |  |
| Compass Review | http://www.hostos.cuny.edu/oaa/compass/algebra_prac5.htm |  |  |

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1.1 Pre-ALGEBRA REVIEW: Do You REMEMBER FRACTIONS?

1. Write a fraction that is equal to $1 / 2$ :
2. Write a fraction that is equal to 1 :
3. Write a fraction that is greater than 1 :
4. Write a fraction that is less than $\frac{1}{5}$ :
5. Write a fraction that is equal to 0 :
6. What is half of $\frac{2}{3}$ ?
7. Write a fraction that is UNDEFINED:

Simplify:
8. $\frac{4}{10}+\frac{3}{10}$
9. $\frac{3}{10}+\frac{2}{15}$
10. $\frac{4}{5} \cdot \frac{15}{9}$
11. $\frac{4}{5} \div \frac{8}{15}$
12. $3 \frac{7}{9}-1 \frac{1}{6}$
13. $5 \frac{1}{2} \cdot 2 \frac{4}{5}$
14. $\left(\frac{3}{4}\right)^{2}$
15. $\frac{15}{3}+\frac{9}{16}\left(1 \frac{1}{3}\right)^{2}$

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 Elementary Algebra Workbook1.2 Pre-Algebra Review: Fractions Video \& Exercise List

| Topic | Website | Videos | Exercises |
| :--- | :--- | :--- | :--- |
| Understanding Fractions | www.khanacademy.org | Numerator, Denominator of a Fraction | Recognizing Fractions 0.5 |
|  |  |  | Recognizing Fractions |
|  |  |  | Fractions on the Number line 1 |
| Equivalent Fractions | www.khanacademy.org | Equivalent Fractions | Fraction Word Problems 1 |
|  |  | Equivalent Fractions Example | Simplifying Fractions |
|  |  | Comparing Fractions | Comparing Fractions 1 |
|  |  | Fractions in Lowest Terms | Equivalent Fractions |
|  |  | Ordering Fractions | Equivalent Fractions 2 |
| Comparing Fractions 2 | Comparing Fractions 2 |  |  |
| Add, Subtract Fractions | www.khanacademy.org | Adding Fractions w/ Like Denominators | Adding Frac. w/ Common Denom |
|  | Subtracting Fractions | Subtract Frac. w/Common Denom |  |
|  |  | Adding and Subtracting Fractions | Adding Fractions |
|  | Adding Fractions w/ unlike denom | Subtracting Fractions |  |
| Multiplying Fractions | Adding Fractions Ex. 1 | Adding and Subtracting Fractions |  |
|  | www.khanacademy.org | Multiplying Fractions | Multiplying Fractions 0.5 |
| Dividing Fractions | Multiplying Fractions Word Problem | Multip. Fractions Word Problems |  |
|  | www.khanacademy.org | Dividing Fractions | Dividing Fractions 0.5 |
|  | Dividing Fractions Example | Dividing Fractions Word Problems |  |
|  |  | Dividing Fractions Word Problems |  |
|  |  |  |  |


| Topic | Website | Videos | Exercises |
| :--- | :--- | :--- | :--- |
| Mixed Numbers and | www.khanacademy.org | Proper and Improper Fractions | Fractions on the Number Line 2 |
| Improper Fractions |  | Comparing Imp Frac \& Mixed Numbers | Comparing Imp Frac \& Mixed No. |
|  |  | Mixed Numbers and Improper Frac. | Converting Mixed Numbers \& I.F. |
|  | Changing a Mixed Number to Imp Frac |  |  |
|  | Changing an Imp Fract to a Mixed No. |  |  |
| Mixed Number Add \& Sub | www.khanacademy.org | Ordering Imp. Fractions \& Mixed No. |  |
|  | Adding Mixed Numbers | Add/Subt Mixed Numbers 0.5 |  |
|  |  | Adding Mixed Nos. w/ Unlike Denom | Add/Subt Mixed Numbers 1 |
|  | Adding Mixed Nos. Word Problem |  |  |
| Mixed Number Mult \& Div |  | Subtracting Mixed Numbers |  |
|  | Subtracting Mixed Numbers 2 |  |  |
|  | Subtracting Mixed Numbers Word Prob |  |  |
|  | Multiplying Fractions and Mixed Nos. | Multiplying Mixed Numbers 1 |  |
|  | Multiplying Mixed Numbers |  |  |
|  |  | Dividing Mixed Numbers |  |

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### 1.3 Pre-Algebra Review: Do You Remember Integers?

SIMPLIFY:

1. $-5+-3$
2. $-5+3$
3. $3-5$
4. $-3-5$
5. $-3-(-5)$
6. $-5-(-3)$
7. $-5-3$
8. $-5(-2)$
9. $-5(2)$
10. $10 \div(-2)$
11. $-10 \div(-2)$
12. $-3(5-4)$
13. $-5^{2}$
14. $(-5)^{2}$
15. $7-6(2-3)^{3}$
16. $14 \div 7(-5+4)^{2}$
17. $\frac{-6}{-12}$

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### 1.4 Pre-Algebra Review: Integers Video \& Exercise List

| Topic | Website | Videos | Exercises |
| :---: | :---: | :---: | :---: |
| Negative Number Basics | www.khanacademy.org | Negative Numbers Introduction | Number Line 2 |
|  |  | Ordering Negative Numbers | Ordering Negative Numbers |
|  |  |  | Number Line 3 |
| Adding Integers | www.khanacademy.org | Example: Adding Negative Numbers | Adding Negative Numbers |
|  |  | Ex: Adding integers w/ diff. signs |  |
| Subtracting Integers | www.khanacademy.org | Why subtracting neg is adding positive | Adding and Subtracting Neg Num. |
|  | www.stlcc.edu | Subtracting Integers PPT on Blackboard |  |
|  |  | Adding/Sub Negative Numbers |  |
| Multiplying/Dividing Neg \# | www.khanacademy.org | Multiplying Pos and Neg Numbers | Mult/Div Negative Numbers |
|  |  | Why Neg x Neg is positive | Negative Number Word Probs |
|  |  | Dividing Pos and Neg Numbers |  |
|  |  | Example: Mult \#'s w/ diff signs |  |
|  |  | Mult and Div Negative numbers |  |
| Absolute Value | www.khanacademy.org | Absolute Value and Number Lines | Finding Absolute Values |
|  |  | Absolute Value 1 | Comparing Absolute Values |
|  |  | Absolute Value of Integers |  |
|  |  | Comparing Absolute Values |  |
| Exponents | www.khanacademy.org | Level 1 Exponents | Positive and Zero Exponents |
|  |  | Understanding Exponents 2 |  |
| Scientific Notation | www.khanacademy.org | Scientific Notation | Scientific Notation |
|  |  | Scientific Notation 1 |  |
| Square Roots | www.khanacademy.org | Understanding Square Roots | Square Roots |

## Adult Learning Academy Elementary Algebra Workbook 1.5 PrActice with Fractions and Integers

Simplify the expression in each box and color any that match the same color.
There should be three of each answer!

| $\frac{4}{5}+\frac{3}{10}$ | -1-1 | $\frac{3}{6}-\frac{4}{8}$ | $6 \div 0$ | $\frac{2}{1}$ |
| :---: | :---: | :---: | :---: | :---: |
| (-5)(4) | $\frac{3}{6}+\frac{5}{10}$ | Half of $1 / 2$ | $-3^{2}$ | Double 10. |
| $-32+32$ | $(-10)^{2}$ | $\frac{47}{0}$ | $\frac{33}{30}$ | Half of $\frac{5}{3}$ |
| $10-(-2)$ | $\frac{-10}{2}$ | $\frac{-50}{-25}$ | -15-5 | $7+(-6)$ |
| -2-7 | $\frac{7}{14} \div 2$ | $\frac{1}{2}+\frac{1}{3}$ | $\frac{0}{47}$ | $\frac{45}{54}$ |
| $15-(-5)$ | $5 \frac{1}{2} \div 5$ | $3 \frac{1}{2} \cdot 1 \frac{1}{2}$ | -4-1 | $-5+3$ |
| $\frac{15}{37} \cdot \frac{37}{15}$ | $-6 \div 3$ | $10^{2}$ | $-11+2$ | $\|3-5\|$ |
| $5 \frac{1}{4}$ | $9 \div(3-3)$ | (-3)(-4) | $\frac{3}{2} \div \frac{2}{7}$ | 0-5 |
| $\frac{5}{2} \cdot \frac{24}{5}$ | (-4)(-5) | $50 \div \frac{1}{2}$ | $-12 \cdot \frac{5}{3}$ | . 25 |

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1.6 SIMPLIFYING Expressions

1. $5 x+3 x$
2. $5(\mathrm{x}-2)$
3. $5 x-3 x$
4. $3(x+1)$
5. $3 x-5 x$
6. $5(\mathrm{x}-1)+3(\mathrm{x}+2)$
7. $\mathrm{x}+\mathrm{x}$
8. $3 x+5-(2 x+1)$
9. $\mathrm{x}-\mathrm{x}$
10. $3 x+5-(2 x-1)$
11. $\mathrm{x} \cdot \mathrm{x}$
12. $3 x+5(2 x-1)$
13. $\mathrm{x} \div \mathrm{x}$
14. $3 x-5(2 x-1)$
15. $x+y$
16. $7-3(2 \mathrm{x}-1)$
17. $3 x+3 y+5 x-y$
18. $7-3(2 \mathrm{x}+1)$

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1.7 Solving 1-Step Equations

1. $\mathrm{x}+3=15$
2. $x-4=20$
3. $6 \mathrm{y}=48$
4. $\frac{a}{3}=12$
5. $\mathrm{w}+100=-300$
6. $\mathrm{x}-12=-20$
7. $-6 y=48$
8. $\frac{a}{3}=-9$
9. $-5=x+4$
10. $5 \mathrm{x}=7$
11. $1 / 2 \mathrm{x}=12$
12. $3 / 4 \mathrm{x}=18$
13. $7 \mathrm{x}=7$
14. $\mathrm{x}-\frac{1}{2}=\frac{3}{2}$
15. $-\mathrm{x}=-7$
16. $5 x=0$

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1.8 Solving 2-STEP EqUATIONS

1. $2 \mathrm{x}+1=7$
2. $3 x-1=11$
3. $-2 \mathrm{x}+1=9$
4. $-5 x-1=9$
5. $5+3 x=17$
6. $7-3 x=13$
7. $7=5+2 x$
8. $10-3 x=13$
9. $\frac{x+4}{3}=10$
10. $\frac{x-7}{5}=2$
11. $-4 \mathrm{a}+2=2$
12. $\frac{w}{3}-10=0$

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1.9 Solving Multi-Step Equations

1. $x+3 x=12$
2. $5 x-3 x+2=12$
3. $5(x-2)=20$
4. $3(x+1)=15$
5. $-2(x+4)=16$
6. $-5 x+3=-4 x$
7. $x-5=2 x$
8. $3 \mathrm{x}=\mathrm{x}+4$
9. $4 \mathrm{x}=2 \mathrm{x}+10$
10. $2(\mathrm{x}+1)=\mathrm{x}-3$
11. $-2(x+1)=3 x-7$

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1.10 Expressions \& Equations

EXPRESSION (SIMPLIFY if possible)

1. $\mathrm{x}+\mathrm{x}+\mathrm{x}$
2. $3(x-4)$
3. $5 \mathrm{x}-\mathrm{x}$
4. $2-\mathrm{x}$
5. $\mathrm{x}-5-3$
6. $7-2(x+1)$
7. $7-2(x-1)$
8. $4 \mathrm{x}-1 / 2 \mathrm{x}$

## EQUATION (SOLVE)

9. $\mathrm{x}+\mathrm{x}+\mathrm{x}=12$
10. $3(x-4)=5$
11. $5 \mathrm{x}-\mathrm{x}=-20$
12. $2-\mathrm{x}=-6$
13. $x-5-3=80$
14. $7-2(x+1)=-1$
15. $7-2(x-1)=-1$
16. $4 \mathrm{x}-1 / 2 \mathrm{x}=7$

## Adult Learning Academy <br> Elementary Algebra Workbook <br> 1.11 SPECIAL EQUATIONS

$x+4=x$
When we subtract x from both sides of this equation, we get $4=0$. The variables have disappeared, and we are left with a FALSE statement. There is NO solution-we have a CONTRADICTION!
$3 x-5=3 x+1$
When we subtract 3 x from both sides of this equation, we get $-5=1$ (or some other false statement with no variables in it). This is also a CONTRADICTION and has no solution!
$4 x+5=4 x+5$
If we subtract 4 x from both sides of this equation, we get $5=5$. The variables have disappeared, but this time the statement we have left is TRUE! This is called an IDENTITY, and ALL real numbers are solutions.
$3(x-6)=3 x-18$
If you try solving this equation, you lose all your variables, and the remaining statement is true. This is also an identity. It doesn't matter what value we put in for x , the statement is always true!

NOTE: These special equations (contradictions and identities) only come up once in a while. Most equations are what we call "Conditional"they have a solution, but not every number in the world is a solution.

What type of equation is it-Identity? Contradiction? Conditional?

1. $3 x+5=5$
2. $3 x+5=3 x+5$
3. $3 x+5=3 x+7$
4. $3(x+2)=3 x+6$
5. $3(x+2)=3 x+5$
6. $3(x+2)=2 x+5$ $\qquad$
7. $\mathrm{x}=\mathrm{x}$
8. $x=x-1$
9. $4 \mathrm{x}+1=\mathrm{x}-2$
10. $x+x=2 x$
11. $\mathrm{x}+\mathrm{x}=3 \mathrm{x}$
12. $x+x=6$
13. $x+x=x+1$
14. $\mathrm{x}+1=\mathrm{x}+2$
15. $x+1=6$

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1.12 SOLVING INEQUALITIES

1. $\mathrm{x}+4>12$
a. Graph:

b. Set-builder $\{\mathrm{X} \mid$
c. Interval notation:
2. $\mathrm{x}+4>12$
a. Graph:

b. Set-builder $\{\mathrm{X} \mid$
\}
c. Interval notation:
3. $\mathrm{x}+4>12$
a. Graph:

b. Set-builder $\{\mathrm{X} \mid$
c. Interval notation:
4. $\mathrm{x}+4>12$
a. Graph:

b. Set-builder $\{\mathrm{X} \mid$
\}
c. Interval notation:
5. $\mathrm{x}+4>12$
a. Graph:

b. Set-builder $\{\mathrm{X} \mid$
\}
c. Interval notation:
6. $\mathbf{x}+4>12$
a. Graph:

b. Set-builder $\{\mathrm{X} \mid$
c. Interval notation:
7. $\mathrm{x}+4>12$
a. Graph:

b. Set-builder $\{\mathrm{X} \mid \quad\}$
c. Interval notation:
8. $\mathrm{x}+4>12$
a. Graph:

b. Set-builder $\{\mathrm{X} \mid \quad\}$
c. Interval notation:
9. $\mathrm{x}+4>12$
a. Graph:

b. Set-builder $\{\mathrm{X} \mid$
c. Interval notation:
10. $\mathrm{x}+4>12$
a. Graph:

b. Set-builder $\{\mathrm{X} \mid$
c. Interval notation:
11. $\mathrm{x}+4>12$
a. Graph:

b. Set-builder $\{\mathrm{X} \mid$
c. Interval notation:
12. $\mathrm{x}+\mathbf{4}>12$
a. Graph:

b. Set-builder $\{\mathrm{X} \mid$
\}
c. Interval notation:

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1.13 Color Matching Inequalities

Find three ways (one from each column) to express the same inequality. Color the matches the same color.

| +1, | $x>3$ | $(-\infty, 3)$ |
| :---: | :---: | :---: |
| $\longleftarrow$ ) | $x<-3$ | $[3, \infty)$ |
| $\longleftarrow$ | $x \geq 3$ | $(-\infty, 3]$ |
| +1, | $-3<x \leq 3$ | $(3, \infty)$ |
| $\longleftarrow)$ | $x \leq-3$ | $(-3,3)$ |
| $\xrightarrow{[+\ldots+3}$ | $-3<x<3$ | $[-3,3)$ |
| $\longleftarrow 1$ | $x \leq 3$ | $(-3,3]$ |
| ( | $-3 \leq x<3$ | $(-\infty,-3)$ |
|  | $x \geq-3$ | $[-3, \infty)$ |
| $(+1+2)$ | $x<3$ | $(-\infty,-3]$ |

## Adult Learning Academy <br> Elementary Algebra Workbook 1.12 CAREER APPLICATIONS: STEM

1. A laptop computer weighs 3 pounds.
a. How much would 2 computers weigh? $\qquad$
b. How much would 10 computers weigh? $\qquad$
c. How much would $x$ computers weigh? $\qquad$
2. An experimental plant has $X$ leaves. Write an expression for the number of leaves on these plants:
a. A plant with two more leaves than the experimental plant: $\qquad$
b. A plant with 4 fewer leaves than the experimental plant: $\qquad$
c. A plant with twice as many leaves as the experimental plant: $\qquad$
d. A plant with 3 more than 5 times as many leaves as the experimental plant: $\qquad$
e. A plant with 1 less than twice as many leaves as the experimental plant: $\qquad$
f. A plant with the same number of leaves as the experimental plant: $\qquad$
3. One solution contains X milliliters of saline. A second solution contains Y milliliters of saline. Write an expression for each situation:
a. If we add 20 ml of saline to the first solution, how much saline will it contain? $\qquad$
b. The total saline used in both solutions: $\qquad$
c. If $\mathrm{X}>\mathrm{Y}$, how much more saline is in the first solution than the second? $\qquad$
d. The mean amount of saline in the two solutions: $\qquad$
e. If we double the amount of saline in each solution, what will be the total amount of saline?
$\qquad$
f. If we pour the first solution equally into four smaller beakers, how much saline will be in each?
$\qquad$
4. Using the variable $R$ to represent the number of millions of red blood cells (RBC) per cubic milliliter of blood. Write an EQUATION for each description (use an = sign!). Then solve.
a. If Charles’ red blood cell count increased by 1.3, his red blood cell count would be at the maximum normal of 6 . What is his current RBC?
b. A change in medication lowered Rhonda's RBC by 2.1 , so it is now 3.9. What was Rhonda's starting RBC?
c. James' RBC started a medical trial at 7.2 and ended at 4.9. How much did his RBC drop during the course of the study?
d. If my RBC doubled, it would be 7. What is it currently?
e. Half of Robert's RBC would be 2.6. What is his current RBC?
f. The total RBC for two brothers is 12. The older brother's RBC is 1.5 higher than the younger brother's RBC. What is the red blood cell count for each brother?
g. The total RBC for two sisters is 9 . The sick sister's RBC is double that of the healthy sister. What is the red blood cell count for each sister?
h. The total RBC for two siblings is 12. The brother's RBC is one less than the twice that of his sister. What is the red blood cell count for each sibling?
5. Write an equation for each situation. Then solve.
a. An updated program requires 40 gigabytes of memory more than its predecessor. The original program required 100 gigabytes. How much does its updated version require?
b. The perimeter of the rectangular lab is 88 feet. The length is 4 feet more than the width. What are the dimensions of the lab?
c. The perimeter of the rectangular computer storage room is 150 feet. The length is twice the width. What are the dimensions of the storage room?
d. Insurance will pay half the replacement cost due to flooded equipment, after the lab pays the $\$ 500$ deductible. The replacement costs $\$ 15,000$. How much will insurance pay?
6. Write an expression for the perimeter and the area of each.
a. X inches

X inches
b.

c. X miles
Perimeter: $\qquad$
Area: $\qquad$
Perimeter: $\qquad$
Area: $\qquad$ Area $\qquad$
7. Expressing Inequalities: Let X equal the volume of liquid to be used in an experiment. Write an inequality to describe each situation.
a. To avoid overloading the equipment, the volume must be at most 10 liters.
b. To ensure accurate measurement, the volume must be at least 3 liters.
c. To conduct the experiment, the volume of liquid must be between 3 and 10 liters, inclusive.
d. If a liter of liquid is lost to condensation, the remaining volume must be at least 3 liters.

### 1.1 Do You Remember Fractions?

1. $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{50}{100}$, etc
2. $\frac{1}{1}, \frac{2}{2}, \frac{37}{37}, \frac{456}{456}$, etc
3. $\frac{9}{8}, \frac{10}{8}, \frac{5}{2}$, numerator $>$ denominator
4. $\frac{1}{6}, \frac{1}{10}, \frac{1}{50}$, etc
5. $\frac{0}{2}, \frac{0}{6}, \frac{0}{329}$, etc
6. of means multiply: $\frac{1}{2} \times \frac{2}{3}=\frac{1}{3}$
7. $\frac{1}{0}, \frac{7}{0}, \frac{387}{0}$,etc
8. $\frac{7}{10}$
9. $\frac{9}{30}+\frac{4}{30}=\frac{\mathbf{1 3}}{\mathbf{3 0}}$
10. $\frac{60}{45}=\frac{4}{3}=1 \frac{1}{3}$
11. $\frac{4}{5} \times \frac{15}{8}=\frac{60}{40}=\frac{3}{2}=1 \frac{1}{2}$
12. $\frac{34}{9}-\frac{7}{6}=\frac{68}{18}-\frac{21}{18}=\frac{47}{18}=2 \frac{\mathbf{1 1}}{18}$
13. $\frac{11}{2} \cdot \frac{14}{5}=\frac{154}{10}=15 \frac{2}{5}$
14. $\frac{3}{4} \cdot \frac{3}{4}=\frac{9}{16}$
15. $5+\frac{9}{16}\left(\frac{4}{3}\right)^{2}=5+\frac{9}{16}\left(\frac{16}{9}\right)=5+1=6$

### 1.3 Do you Remember Integers?

1. -8
2. -2
3. -2
4. -8
5. 2
6. -2
7. -8
8. 10
9. -10
10. -5
11. 5
12. -3
13.     - 25
14. 25
15. $7-6(-1)^{3}=7-6(-1)=7+6=13$
16. $14 \div 7(-1)^{2}=14 \div 7(1)=2$
17. $\frac{1}{2}$
1.5 Practice with Fractions \& Integers

| $\begin{aligned} & \frac{4}{5}+\frac{3}{10} \\ & =1 \frac{1}{10} \\ & \hline \end{aligned}$ | $\begin{gathered} -1-1 \\ =-2 \end{gathered}$ | $\begin{gathered} \frac{3}{6}-\frac{4}{8} \\ =0 \end{gathered}$ | $\begin{gathered} 6 \div 0 \\ \text { undefined } \end{gathered}$ | $\begin{gathered} \frac{2}{1} \\ =2 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} (-5)(4) \\ =-20 \end{gathered}$ | $\frac{3}{6}+\frac{5}{10}$ | $\begin{aligned} & \text { Half of } 1 / 2 \\ & =1 / 4 \end{aligned}$ | $\begin{aligned} & -3^{2} \\ & =-9 \end{aligned}$ | $\begin{gathered} \text { Double } 10 . \\ =\mathbf{2 0} \end{gathered}$ |
| $\begin{gathered} -32+32 \\ =0 \end{gathered}$ | $\begin{gathered} (-10)^{2} \\ =100 \end{gathered}$ | $\frac{47}{0} \text { undefined }$ | $\begin{array}{r} \frac{33}{30} \\ 1 \frac{1}{10} \end{array}$ | Half of $\frac{5}{3}$ $=\frac{5}{6}$ |
| $\begin{gathered} 10-(-2) \\ =12 \end{gathered}$ | $\frac{-10}{2}$ $=-5$ | -50 <br> -25 <br> $=2$ | $\begin{gathered} -15-5 \\ =-20 \end{gathered}$ | $\begin{gathered} 7+(-6) \\ =\mathbf{1} \end{gathered}$ |
| $\begin{gathered} -2-7 \\ =-9 \end{gathered}$ | $\frac{7}{14} \div 2$ $=1 / 4$ | $\begin{gathered} \frac{1}{2}+\frac{1}{3} \\ =\frac{5}{6} \end{gathered}$ | $\begin{aligned} & \frac{0}{47} \\ & =0 \end{aligned}$ | $\begin{array}{r} \frac{45}{54} \\ =\frac{5}{6} \end{array}$ |
| $\begin{gathered} 15-(-5) \\ =\mathbf{2 0} \end{gathered}$ | $\begin{gathered} 5 \frac{1}{2} \div 5 \\ 1 \frac{1}{10} \end{gathered}$ | $\begin{aligned} & 3 \frac{1}{2} \cdot 1 \frac{1}{2} \\ & =5 \frac{1}{4} \\ & \hline \end{aligned}$ | $-4-1$ $=-5$ | $\begin{gathered} -5+3 \\ =-2 \end{gathered}$ |
| $\frac{15}{37} \cdot \frac{37}{15}$ | $\begin{gathered} -6 \div 3 \\ =-2 \end{gathered}$ | $\begin{gathered} 10^{2} \\ =100 \end{gathered}$ | $\begin{gathered} -11+2 \\ =-9 \end{gathered}$ | $\begin{gathered} \|3-5\| \\ =\mathbf{2} \end{gathered}$ |
| $5 \frac{1}{4}$ | $\begin{gathered} 9 \div(3- \\ 3) \\ \text { undefined } \\ \hline \end{gathered}$ | $\begin{gathered} (-3)(-4) \\ =12 \end{gathered}$ | $\begin{gathered} \frac{3}{2} \div \frac{2}{7} \\ =5 \frac{1}{4} \end{gathered}$ | $\begin{aligned} & 0-5 \\ & =-5 \end{aligned}$ |
| $\begin{gathered} \frac{5}{2} \cdot \frac{24}{5} \\ =12 \end{gathered}$ | $\begin{gathered} (-4)(-5) \\ =\mathbf{2 0} \end{gathered}$ | $\begin{aligned} & 50 \div \frac{1}{2} \\ & =100 \end{aligned}$ | $\begin{gathered} -12 \cdot \frac{5}{3} \\ =-20 \end{gathered}$ | $\begin{gathered} .25 \\ =\mathbf{1} / \mathbf{4} \end{gathered}$ |

### 1.6 Simplifying Expressions

1. 8 x
2. $2 x$
3. $-2 x$
4. 2 x
5. 0
6. $\mathrm{x}^{2}$
7. 1
8. $x+y$ (not like terms)
9. $8 x+2 y$
10. $5 x-10$
11. $5 x-5+3 x+6=8 x+1$
12. $3 x+5-2 x-1=x+4$
13. $3 x+5-2 x+1=x+6$
14. $3 x+10 x-5=13 x-5$
15. $3 x-10 x+5=-7 x+5$
16. $7-6 x+3=10-6 x$
17. $7-6 x-3=4-6 x$

### 1.7 One-Step Equations

1. $x+3-3=15-3$
$\mathbf{x}=12$
2. $x-4+4=20+4$

$$
x=24
$$

3. $6 y / 6=48 / 6 \quad y=8$
4. $\left(\frac{a}{3}\right) 3=(12) 3$

$$
a=36
$$

5. $\mathrm{w}+100-100=-300-100$

$$
w=-400
$$

6. $x-12+12=-20+12$

$$
x=-8
$$

7. $-6 y /-6=48 /-6$ $y=-8$
8. $\left(\frac{a}{3}\right) 3=(-9) 3$
$a=-27$
9. $-5-4=x+4-4$
$-9=x$ or $x=-9$
10. $5 x / 5=7 / 5$
$x=7 / 5$
11. $(1 / 2 x) 2 / 1=(12) 2 / 1$ $x=24$
12. $(3 / 4 \mathrm{x}) 4 / 3=(18) 4 / 3$

$$
\mathbf{x}=72 / 3=24
$$

13. $7 x / 7=7 / 7 \quad x=1$
14. $\mathrm{x}-\frac{1}{2}+\frac{1}{2}=\frac{3}{2}+\frac{1}{2}$

$$
\mathbf{x}=\frac{4}{2}=2
$$

15. $-x /-1=-7 /-1 \quad x=7$
16. $5 x / 5=0 / 5 \quad x=0$

### 1.8 Two-Step Equations

$$
\begin{aligned}
& \text { 1. } 2 x+1-\mathbf{1}=7-\mathbf{1} \\
& 2 x=6 \\
& 2 x / 2=6 / 2 \quad x=\mathbf{3}
\end{aligned}
$$

2. $3 x-1+1=11+1$
$3 \mathrm{x}=12$
$3 x / 3=12 / 3$
$\mathrm{x}=4$
3. $-2 x+1-\mathbf{1}=9-1$
$-2 x=8$
$-2 \mathrm{x} /-2=8 /-2$
$x=-4$

### 1.8 Two-Step Equations (cont.)

4. $-5 x-1+1=9+1$
$-5 x=10$
$-5 x /-5=10 /-5$
$\mathrm{x}=-2$
5. $5+3 x-5=17-5$
$3 \mathrm{x}=12$
$3 x / 3=12 / 3$
$\mathrm{x}=4$
6. $7-3 x-7=13-7$
$-3 x=6$
$-3 \mathrm{x} /-3=6 /-3$
$x=-2$
7. $7-5=5+2 x-5$
$2=2 \mathrm{x}$
$2 / 2=2 x / 2$
$1=x$ or $x=1$
8. $10-3 x-10=13-10$
$-3 x=3$
$-3 x /-3=3 /-3$
$\mathrm{x}=-1$
9. $\left(\frac{x+4}{3}\right) 3=(10) 3$
$x+4-4=30-4$
$\mathrm{x}=26$
10. $\left(\frac{x-7}{5}\right) 5=(2) 5$
$x-7+7=10+7$
$\mathrm{x}=17$
11. $-4 a+2-2=2-2$
$-4 a=0$
$-4 a /-4=0 /-4$
$\mathbf{a}=\mathbf{0}$
12. $\frac{w}{3}-10+10=0+10$
$\left(\frac{w}{3}\right) 3=(10) 3$
$\mathrm{w}=30$

### 1.9 Multi-Step Equations

1. $x+3 x=12$
$4 x=12$
$4 \mathrm{x} / 4=12 / 4$
$\mathrm{x}=3$
2. $5 x-3 x+2=12$
$2 x+2=12$
$2 \mathrm{x}+2-2=12-2$
$2 \mathrm{x}=10$
$2 \mathrm{x} / 2=10 / 2$
$x=5$

### 1.9 Multi-Step Equations

3. $3 x-5 x+2=12$
$-2 x+2=12$
$-2 \mathrm{x}+2-2=12-2$
$-2 x=10$
$-2 \mathrm{x} /-2=10 /-2$
$x=-5$
4. $5(\mathrm{x}-2)=20$
$5 x-10=20$
$5 \mathrm{x}-10+\mathbf{1 0}=20+\mathbf{1 0}$
$5 \mathrm{x}=30$
$5 x / 5=30 / 5$
$x=6$
5. $3(x+1)=15$
$3 x+3=15$
$3 \mathrm{x}+3-3=15-3$
$3 \mathrm{x}=12$
$3 x / 3=12 / 3$
$x=4$
6. $-2(x+4)=16$
$-2 x-8=16$
$-2 \mathrm{x}-8+8=16+8$
$-2 \mathrm{x}=24$
$-2 x /-2=24 /-2$
$\mathrm{x}=-12$
7. $3 \mathrm{x}-\mathrm{x}=\mathrm{x}+4-\mathrm{x}$
$2 \mathrm{x}=4$
$2 x / 2=4 / 2$
$\mathrm{x}=2$
8. $4 \mathrm{x}-2 \mathrm{x}=2 \mathrm{x}+10-2 \mathrm{x}$
$2 \mathrm{x}=10$
$2 \mathrm{x} / \mathbf{2}=10 / 2$
$\mathrm{x}=5$
9. $-5 x+3+5 x=-4 x+5 x$

$$
3=1 \mathrm{x} \quad \text { or } \quad \mathbf{x}=3
$$

10. $x-5-x=2 x-x$
$-5=x$ or $x=-5$
11. $2(x+1)=x-3$ $2 x+2=x-3$
$2 x+2-2=x-3-2$
$2 \mathrm{x}=\mathrm{x}-5$
$2 \mathrm{x}-\mathrm{x}=\mathrm{x}-5-\mathrm{x}$
$x=-5$
12. $-2(x+1)=3 x-7$
$-2 x-2=3 x-7$
$-2 \mathrm{x}-2+7=3 \mathrm{x}-7+7$
$-2 x+5=3 x$
$-2 \mathrm{x}+5+2 \mathrm{x}=3 \mathrm{x}+2 \mathrm{x}$
$5=5 x$
$5 / 5=5 x / 5$
$1=\mathrm{x}$ or $\mathrm{x}=\mathbf{1}$
1.10 Expressions and Equations
13. 3 x
14. $3 \mathrm{x}-12$
15. 4 x
16. $2-\mathrm{x}$ (not like terms)
17. $\mathrm{x}-8$
18. $7-2 x-2$ $=5-2 x$
19. $7-2 \mathrm{x}+2$ $=9-2 x$
20. $3^{1 / 2 x}$ or $3.5 x$
21. $3 x=12$

$$
x=12 / 3=4
$$

10. $3 x-12=5$
$3 x=17$
$x=17 / 3=52 / 3$ or 5.666
11. $4 \mathrm{x}=-20$
$x=-20 / 4=-5$
12. $-x=-8$
$-x /-1=-8 /-1 \quad$ so $\mathbf{x}=8$
13. $\mathrm{x}-8=80$ so $\mathrm{x}=\mathbf{8 8}$
14. $7-2 x-2=-1$
$5-2 x=-1$
$-2 x=-6$
$\mathbf{x}=-6 /-2=3$
15. $7-2 x+2=-1$
$9-2 x=-1$
$-2 \mathrm{x}=-10$
$\mathbf{x}=-10 /-2=5$
16. $3.5 x=7$
$\mathbf{x}=7 / 3.5=2$

### 1.11 Special Equations

1. $\mathrm{x}=0$ conditional
2. identity
3. contradiction
4. identity
5. contradiction
6. $x=-1$ conditional
7. identity
8. contradiction
9. $x=-1$ conditional
10. identity
11. $\mathrm{x}=0$ conditional
12. $x=3$ conditional
13. $x=1$ conditional
14. contradiction
15. $\mathrm{x}=5$ conditional

### 1.12 Solving Inequalities

1 a.


1b. $\{x \mid x>8\}$
1c. $(8, \infty)$

2a.


2b. $\{x \mid x \leq 5 \quad\}$
2c. $(-\infty, 5]$

3a.


3b. $\{x \mid x \geq 6\}$
3c. $[6, \infty)$

4a.


4b. $\{x \mid x<-3\}$ don't forget to FLIP!
4c. $(-3, \infty)$
$5 a$.


5b. $\{x \mid x<4\}$
5c. $(-\infty, 4)$

6 .


6b. $\{x \mid 3<x<5\}$ work with all 3 sections!
6c. $(3,5)$

7 a.


7b. $\{x \mid x<-4\}$
7c. $(-\infty,-4)$

8a.


8b. $\{x \mid x \leq 2\}$
8c. $(-\infty, 2]$

9a.
9b. $\left\{x \mid-\mathbf{1}^{-15} \leq x<2\right\}$
9c. $[-1,2)$

10a.


10b. $\{x \mid x>5 / 2\}$
10c. $\left(\frac{5}{2}, \infty\right)$

### 1.12 Solving Inequalities (cont.)

11a.


11b. $\{x \mid x<12\}$
11c. $(-\infty, 12)$

12a.


12b. $\{x \mid-5<x<-3\}$
12c. $(-5,-3)$
1.13 Color Matching Inequalities

1.14 Career Pathway Applications -STEM (cont.)

1a. $3 \cdot 2=6$ pounds
1b. $3 \cdot 10=\mathbf{3 0}$ pounds
1c. $3 \cdot x=3 x$ pounds

2a. $x+2$
2b. $x-4$
2c. $2 x$
2d. $5 x+3$
2e. $2 x-1$
2f. $2 x-1$

3a. $x+20$ or $20+x$
3b. $x+y$
3c. $x-y$
3d. $1 / 2(x+y)$ or $\frac{x+y}{2}$
3e. $2 x+2 y$ or $2(x+y)$
3f. $\mathbf{x} / 4$ or $1 / 4 x$ or $.25 x$

4a. $R+1.3=6$
$\mathrm{R}+1.3$ - $\mathbf{1 . 3}=6$ - $\mathbf{1 . 3}$
R $=4.7$
4b. $R-2.1=3.9$
$R-2.1+2.1=3.9+2.1$
$\mathbf{R}=5$
4c. $7.2-\mathrm{R}=4.9$
$7.2-\mathrm{R}-4.9+\mathbf{R}=4.9-4.9+\mathbf{R}$
$\mathbf{R}=2.3$
4d. $2 R=7$
$2 R / 2=7 / 2$
R = $3^{1 / 2}$ or 3.5
4e. $.5 R=2.6$
$.5 R / .5=2.6 / .5$
$\mathbf{R}=5.2$
4f. let $\mathrm{R}=$ the younger brother's $\mathrm{RBC}=5.25$
So $\mathrm{R}+1.5$ = older brother's $\mathrm{RBC}=\mathbf{6 . 7 5}$
R + R + 1.5 = 12
$2 R+1.5-\mathbf{1 . 5}=12-1.5$
$2 R / \mathbf{2}=10.5 / \mathbf{2} \quad \mathbf{R}=5.2$
4g. let $\mathrm{R}=$ the healthy sister's $\mathrm{RBC}=3$
So 2 R = the sick sister's $\mathrm{RBC}=\mathbf{6}$
$\mathbf{R}+2 \mathbf{R}=\mathbf{9}$
$3 R / \mathbf{3}=9 / \mathbf{3} \quad \mathbf{R}=\mathbf{3}$

### 1.14 Career Pathway Applications -STEM (cont.)

4h. let $\mathrm{R}=$ the sister's $\mathrm{RBC}=41 / 3$ or 4.333
So $2 R-1$ = the brother's RBC $=72 / 3$ or 7.667

$$
R+2 R-1=12
$$

$$
3 R-1+\mathbf{1}=12+\mathbf{1}
$$

$$
3 R / 3=13 / 3 \quad R=41 / 3 \text { or } 4.333
$$

$$
2 \mathrm{R} / 2=10.5 / 2 \quad \mathbf{R}=5.25
$$

5a. let $\mathrm{x}=$ the updated program $=5.25$

$$
x=100+40=140 \text { gigabytes }
$$

5b. Perimeter $=$ sum of all sides (rectangle has 4 sides)
let width $=\mathrm{W}$ and let length $=\mathrm{W}+4$
$W+W+W+4+W+4=88$
$4 W+8-\mathbf{8}=88-\mathbf{8}$
$4 \mathrm{~W} / 4=80 / 4 \quad \mathrm{~W}=20$
width $=\mathbf{2 0 f t}$. length $=\mathbf{2 4 f t}$.
5b. let width $=\mathrm{W}$ and let length $=2 \mathrm{~W}$

$$
\begin{aligned}
& W+W+2 W+2 W=150 \\
& 6 W / 6=150 / 6 \quad W=25 \\
& \text { width }=25 f t . \text { length }=50 \mathrm{ft} .
\end{aligned}
$$

5b. let $\mathrm{x}=$ the amount insurance will pay

$$
\begin{aligned}
& x=1 / 2(15,000-500) \\
& x=1 / 2(14,500)=\$ 7,250
\end{aligned}
$$

Perimeter $=$ sum of all sides; Area $=$ Length $x$ Width
6a. $P=4 x$ inches $\quad A=x^{2}$ inches
6b. $P=4 x+6$ feet $A=x^{2}+3$ sq. feet
6c. $P=6 x$ miles $\quad A=2 x^{2}$ miles

7a. $x \leq 10$
7b. $x \geq 3$
7c. $3 \leq x \leq 10$
7d. $3 \leq x-1 \leq 10$ or $4 \leq x \leq 11$

Adult Learning Academy<br>Elementary Algebra Workbook

Module 2: Applications of Linear equations in one variable, FUNCTION NOTATION

Learning Objectives
By the time you finish this module, you should be able to:
$\square$ Translate verbal expressions to the symbolic language of algebra
$\square$ Use a variety of problem-solving strategies.
$\square$ Solve word problems about unknown numbers that are related to each other.
$\square$ Use a given formula to solve problems.
$\square$ Solve a given formula to isolate a specified variable.Solve distance problems using the formula D = RT
$\square$ Solve problems involving simple interest using the formula I = PRT
$\square$ Solve problems involving ratios and proportions.
$\square$ Solve percent problems.
$\square$ Solve mixture problems.Find the perimeter of a rectangle.
$\square$ Use function $\mathrm{f}(\mathrm{x})$ notation to plug a value in for x .

## IMPORTANT INFORMATION FROM MODULE 2:

Consecutive integers: $\mathrm{x}, \mathrm{x}+1, \mathrm{x}+2, \mathrm{x}+3$, etc.
Consecutive odd or consecutive even integers: $x, x+2, x+4$, etc.
To DOUBLE a number means to multiply it by 2 (you might write 2 x )
Translations: "of" means "multiply"; "is" means "equal"
To solve a proportion, cross multiply.
$\mathrm{D}=\mathrm{rt} \quad$ (distance $=$ rate x time)
$\mathrm{I}=\mathrm{PRT}$ (interest = principal x rate x time)
Perimeter of a rectangle $=2 \mathrm{~L}+2 \mathrm{~W}$
Strategies: Draw a diagram! Look for key words. Read the problem several times. Write a variable for the unknown quantity you wish to find. Try some simple numbers to get a feel for what's happening. Make a guess and adjust. Make a table. See what happens one year at a time, one hour at a time, etc. Notice patterns.
$f(x)$ is pronounced " $f$ of $x$ ". It means to plug the given number in for $x$. For example, $f(3)$ means to plug the number 3 in wherever you see an $x$ in the function, and calculate the answer.

## Adult Learning Academy <br> Elementary Algebra Workbook <br> Module 2 Video \& Exercise List

| Topic | Website | Videos | Exercises |
| :---: | :---: | :---: | :---: |
| Expressions | www.khanacademy.org | Evaluate an Expression | Evaluating expressions in 1 var. |
| and Formulas |  | Evaluate a Formula using Substitution |  |
|  |  | Age Word Problems 1 |  |
| Various Word | www.khanacademy.org | Basic Linear Equation Word Problems | Linear Equation Word Problems |
| Problem Examples | http://www.youtube.com/watch?v=JNusKkf3wbI\&list=PLBUJyzEUI_hyrLI_kbMbWmyKNuZOJ5Nf |  |  |
|  | $\underline{\text { http://www.youtube.com/watch?v=Qv0vF0rH1aM\&list=PLBUJyzEUI hyrLI kbMbWmyKNuZOJ5Nf }}$ |  |  |
|  | http://www.youtube.com/watch?v=bJC4dqEW1gQ\&list=PLBUJyzEUI_hyrLI_kbMbWmyKNuZOJ5Nf_ |  |  |
|  | http://www.youtube.com/watch?v=FG3bwWhkHTU\&list=PLBUJyzEUI hyrLI_kbMbWmyKNuZOJ5Nf |  |  |
| Simple Interest | $\underline{\text { http://www.youtube.com/watch?v=r3-lyBGIJ98 }}$ | The Simple Interest Formula |  |
| Ratio and Proportion | www.khanacademy.org | Introduction to Ratios | Expressing Ratios as Fractions |
|  |  | Ratios as Fractions in Simplest Form | Ratio Word Problems |
|  |  | Simplifying Rates and Ratios | Writing Proportions |
|  |  | Writing Proportions | Proportions1 |
|  |  | Understanding Proportions |  |
| Percent Problems | www.khanacademy.org | Representing \# as Dec, \%, Fraction | Converting Percents to Dec |
|  |  | Growing by a Percentage | Discount Tax and Tip |
|  |  | Solving Percent Problems | Markup, Commission |
|  |  | Solving Percent Problems 2 |  |
| Use Proportions | http://www.youtube.com/watch?v=yl0Rb6T09VM |  |  |
| Use Equations | http://www.youtube.com/watch? $\mathrm{v}=$ LkTYkHbUiU4 |  |  |
| Mixture Problems | www.stlcc.edu | Powerpoint: Mixture Problems |  |
|  | http://www.youtube.com/watch?v=gD2YfPU-qMM |  |  |
|  | http://www.youtube.com/watch?v=DwEsBCHM_jc |  |  |
| Functions | http://www.youtube.com/watch?v=VUTXsPFx-qQ |  |  |
| Function Notation | http://www.youtube.com/watch?v=S1uAy5vM4HI |  |  |
|  | www.khanacademy.org | Understanding Function Notation Ex 1 |  |


| Topic | Website | Videos | Exercises |
| :--- | :--- | :--- | :--- | :--- |
| Module 2 Test Review | http://www.youtube.com/watch?v=WQYzOpcnWxs | Helpful System to Solve Word Problems |  |
|  | $\underline{\text { stlcc.edu }}$ |  | Mixture Problem PowerPoint |
|  |  |  | Module 2 Review Flashcards |

Before you start to panic at the thought of an entire module dedicated to the dreaded word problem, consider this:


The whole point of studying math is to be able to USE it to solve real-life problems.
Real-life situations are expressed in WORDS.

So the goal of math is to be able to solve word problems!
This module will allow you to practice solving the equations from module 1, but in the context of reallife situations. We use the symbols of math to model what's going on in the problem.


There are many tools you can use to help you approach word problems. Not every tool helps with every problem. There are many ways to approach every problem. The more tools you have, the easier the job becomes. Here are some tips:

* Draw a picture! You don't have to be an artist. Diagrams are especially useful for problems involving distance, mixtures, area, and perimeter. If you can picture it, you can solve it!

* Try some simple numbers! You can often get a feel for a problem by using your own "easy" numbers just to see how they are related. For example, say that a problem asks how many hours it will take to travel 37.5 miles at 22 miles per hour. If you don’t know where to start, consider how long it would take to drive 100 miles at 50 miles per hour-that would take 2 hours (the first 50 miles would take one hour, the next 50 miles would take another hour). How can you use 100 and 50 to get 2? Divide! Then go back to the original problem and divide 37.5 by 22.
* Take a guess! Some people feel like they are "cheating" if they try to guess the answer, but it can be a great way to start. For example, say you are trying to find two numbers that add up to 99 , and one number is twice as big as the other. You might guess that the numbers are 30 and 60 , since 60 is twice as big as 30 . Those don't add up to 99 , but they are close. They are just a little too small. Don't give up because your guess wasn’t perfect-adjust your guess! 31 + 62 would be better, but still a bit too small. $32+64$ would be even closer, but still too small. How about $33+66$ ? You've got it!
* Replace words with algebra! For example, in the problem above, you were looking for two numbers whose sum is 99, and one number is twice the other. You might write the following:

| $\begin{array}{c}\text { One } \\ \text { number }\end{array}$ |
| :---: |
| $+\begin{array}{c}\text { The other } \\ \text { number }\end{array}$ |
| $=99$ |

In Algebra, X stands for a number we don't know yet. Replace the number in the first box with X . The "other number" is twice as big as X , so call it 2X. This gives us
$X+2 X=99$. You can solve the equation and find out that $X=33$.


* Remember a similar problem! Try to notice the overall structure of the problems you solve, so you can apply what you've learned to new problems. A problem about a boat instead of a car, or a lady named Gertie instead of a guy named George, or two animals approaching each other instead of leaving each other, or a shopkeeper mixing coffee beans instead of a pharmacist mixing medicines, could be the same basic problem, solved the same way!

* Go one step at a time! In a distance problem, what's happening after one hour? After two hours? Three? In an interest problem, how much interest do you have after one year? Two years? Three? If you work step by step, you'll start to see a pattern!
* Check your answer! Did you just say that a lady is 325 years old? Or you need 3.27 buses, or a rectangle is -5 feet long? Did a number that was supposed to be larger get smaller? Then chances are you did not solve the problem correctly! Use the common sense that is built into your wonderful brain! Always label your answer and read the question one last time to be sure you answered the question asked, and that your answer is reasonable.

[^0]Adult Learning Academy Elementary Algebra Workbook<br>2.2 Formulas

A formula is just an EQUATION with more than one VARIABLE. Formulas express how different amounts are related. As an example, we'll use the distance formula:
$\mathbf{D}=\mathbf{r t} \quad$ This formula tells you that the distance you travel is equal to your rate
(speed), multiplied by the time you travel. You use this formula without even thinking about it. For example, if you have been driving for 2 hours at 60 miles per hour, you know that you have driven 120 miles.

The formula has three variables: $\mathbf{D}$ to represent Distance, $\mathbf{r}$ to represent rate (speed), and $\mathbf{t}$ to represent time. Any word problem involving this formula will give you values to plug in for TWO of the three variables, and will leave one for you to figure out.

Examples: A car travels at 50 miles per hour for 3 hours. How far does it travel? Here, you know the rate and the time, and must find the distance:

$$
\mathrm{D}=50 \cdot 3=150 \text { miles } .
$$

How long will it take to drive 100 miles at a rate of 20 miles per hour? Here, you know the distance and the rate, and must find the time:

$$
100=20 \cdot t \text {, so } t=5 \text { hours. }
$$

I rode my bike for 2 hours and went 20 miles. How fast was I riding? Here, you know the distance and the time, and must find the rate:

$$
20 \text { = r } \cdot 2 \text {, so r = } 10 \text { miles per hour. }
$$

If a formula has 100 variables (don't worry, there are none of those here!), the problem will have to give you values to plug in for 99 of those variables. You'll always have just ONE missing number to find.

Solving a Formula for a particular variable: some problems don’t give you any numbers. They just ask you to "solve for" x, or "solve for" some other variable. DO NOT JUST INVENT SOME NUMBERS! The problem is asking you to write the formula with the specified variable all alone on one side of the equation.

Examples: Solve for r: D = rt
We want the r to be alone. The way the formula is written, the r is multiplied by t . To undo that multiplication, divide both sides of the equation by t . This makes our formula $\frac{D}{t}=r$. The r is isolated. Mission accomplished.

Solve this formula for $\mathrm{y}: ~ 3 \mathrm{x}+\mathrm{y}=7$
We want the y to be alone. The way the formula is written, the y has 3 x added to it. To undo that addition, subtract $3 x$ from both sides of the equation. This makes our formula $y=7-3 x$. The $y$ is isolated. Mission accomplished.

## Elementary Algebra Workbook

2.3 Function Notations

You can think of a function as a machine. Every function machine takes whatever number you give it
input, works with that number, and gives you back a number as output. Consider this function, whose job is to always add 1 :

If we stick a 5 into the function machine, it adds 1 and spits out a 6 . We write $f(5)=6$ and we say " $f$ of 5 equals 6 ".


If we stick a 99 into the function machine, it adds 1 and spits out a 100 . We write $f(99)=100$ and we say "f of 99 equals 100".

If we stick a 3.7 into the function machine, it adds 1 and spits out a 4.7. We write $f(3.7)=4.7$, and we say " $f$ of 3.7 equals 4.7".

In other words, we can input any value of x we choose, and a new number will come out, always according to the same rule.

Here is another function machine. This one takes whatever number we input, multiplies it by 2, and subtracts 3:

If we input a 5 , the machine will take that 5 , multiply it by 2 ,
 and subtract 3 :
$f(5)=2 \cdot 5-3=10-3=7$.
We write $f(5)=7$, and we say that " $f$ of 5 equals 7 ".
If we input a 100 , the machine will take that 100 , multiply it by 2 , and subtract 3 :
$f(100)=2 \cdot 100-3=200-3=197$.
We write $f(100)=197$, and we say that " $f$ of 100 equals 197 ".

## 1. PRACTICE:

a. What is $f(20)$ ?
b. What is $f(0)$ ?
c. What is $\mathrm{f}(-50)$ ?

Of course, we don't always have to draw the function machine. And not every machine has to be called " f ". Let's say we have these functions:

$$
\mathrm{f}(\mathrm{x})=\mathrm{x}+7 \quad \mathrm{~g}(\mathrm{x})=2 \mathrm{x}+5 \quad \mathrm{~h}(\mathrm{x})=\mathrm{x}^{2}
$$

Let's pick some values for x . Let's see what each function does when x is 3 :

$$
\begin{array}{l|l|l}
\mathrm{f}(3)=3+7 & \mathrm{~g}(3)=2 \cdot 3+5 & \mathrm{~h}(3)=3^{2} \\
\mathrm{f}(3)=10 & \mathrm{~g}(3)=11 & \mathrm{~h}(3)=9
\end{array}
$$

Now we'll see what each function does if the x is 0 :
$f(0)=0+7$
$g(0)=2 \cdot 0+5$
$h(0)=0^{2}$
$f(0)=7$
$g(0)=5$
$h(0)=0$
2. PRACTICE: Use the same functions above to calculate the following:
a. $f(2)$
d. $g(2)$
g. $h(2)$
b. $f(-10)$
e. $g(-10)$
h. $h(-10)$
c. $f(8)$
f. $g(8)$
i. $\mathrm{h}(8)$

Adult Learning Academy
Elementary Algebra Workbook
2.4 CAREER APPLICATIONS: STEM

## I. Writing Algebraic Expressions

Use the given information to change the initial number of bees in the colony as described:

1. Environmental researchers have been observing a colony of $\mathbf{5 0 , 0 0 0}$ bees
a. If 10,000 bees joined the colony, how many would there be?
b. If 5,000 of the original bees died, how many would be in the colony?
c. Double the original number of bees:
d. Cut the original number of bees in half:
e. Triple the original number of bees:
f. Decrease the number of bees by 100 :
g. How many is 200 less than half the original number of bees?
h. How many is 50 more than twice the original number of bees?
i. What is the total number of bees in a dozen colonies this size?
j. The colony produces 5 liters of honey. How much honey is this per bee?
2. Researchers cannot tell how many bees are in the colony initially. Use the variable $X$ to represent the number.
a. If 10,000 bees joined the colony, how many would there be?
b. If 5,000 of the original bees died, how many would be in the colony?
c. Double the original number of bees:
d. Cut the original number of bees in half:
e. Triple the original number of bees:
f. Decrease the number of bees by 100:
g. How many is 200 less than half the original number of bees?
h. How many is 50 more than twice the original number of bees?
i. What is the total number of bees in a dozen colonies this size?
j. The colony produces 5 liters of honey. How much honey is this per bee?

## II. Find the secret numbers!

Be sure to set up an equation, identify your variable, label your answer, and check that you answered the question and your answer makes sense!
3. There are 75 patients on a maternity ward. The number of mothers is twelve less than twice the number of babies. How many mamas and how many babies are on the ward?
4. A pair of twins weigh a total of 63 pounds. Their weights are consecutive integers. How much does each twin weigh?
5. A pair of twins weigh a total of 158 pounds. Their weights are consecutive even integers. How much does each twin weigh?
6. A set of triplets weigh a total of 117 pounds. Their weights are consecutive odd integers. How much does each triplet weigh?
7. The perimeter of a laboratory is 44 feet. The length and width are consecutive even integers. What are the dimensions of the room? (Remember, to find perimeter, add up the lengths of all four sides!)
8. There are 132 researchers at a workshop. Some are biologists, while the rest are chemists. The number biologists is twelve less than twice the number of chemists. How many biologists and how many chemists are at the workshop?
9. Two studies would have taken exactly the same amount of time, but because of an equipment malfunction, one had to extend its observations for 5 days. Altogether, the two studies lasted 195 days. How long was each study?
10. According to GPS, three routes to the same destination have distances that are consecutive odd integers. Driving all three routes would make a total of 117 miles. How far is the most direct route?
11. Three plants have a total height of 83 centimeters. The first is 3 centimeters shorter than the second. The third is 5 centimeters taller than the second. How tall is each plant?
12. According to the Tornado History Project (www.tornadohistoryproject.com), the total number of tornadoes in Missouri in 2012, 2013, and 2014 was 125. The number in 2012 was 20 less than the number in 2013. The number in 2014 was two less than the number in 2013. How many tornadoes hit Missouri each year?
13. Andre spent a total of $\$ 1880$ to buy a new i-Pad mini and a new MacBook Pro. The MacBook cost $\$ 290$ more than twice times the cost of the i-Pad. How much did each cost?

## III. WORKING WITH FORMULAS

14. Kinetic Energy Formula

Kinetic Energy, or KE, is energy of motion. If the mass and velocity of a body are known, the kinetic energy can be determined by this formula:

$$
\begin{aligned}
\mathbf{K E}=\frac{\mathbf{m} \mathbf{V}^{2}}{\mathbf{2}} & \begin{array}{l}
\text { where } \mathrm{KE}=\text { Kinetic Energy in Joules } \\
\mathrm{m}=\text { mass in kilograms } \\
\mathrm{V}=\text { velocity in meters } / \mathrm{sec}
\end{array}
\end{aligned}
$$

a. Calculate the kinetic energy of a brown bear with a mass of 317 kg that runs at 15.5 meters $/ \mathrm{sec}$.
b. The Kinetic Energy of a $650-\mathrm{kg}$ roller coaster car is 105,300 Joules. How fast is the car rolling?
c. The kinetic energy of a boat is 52,000 Joules. The boat is moving at 1.63 meters/sec. What is the boat's mass?
d. The Lockheed SR-71 Blackbird, a high-altitude spy plane, has a mass of $77,000 \mathrm{~kg}$, and at top speed it has a kinetic energy of $37,000,000,000 \mathrm{~J}$. What is the top speed of the SR-71?
e. How fast is a 65-kilogram skydiver falling if her kinetic energy is 704,000 Joules?
f. Solve the formula above for mass (that is, get the variable $m$ by itself on one side of the equation).

## 15. Temperature Conversion Formulas

In the United States, most people use a Fahrenheit temperature scale. Along with the rest of the world, however, the Federal Aviation Administration (FAA) uses the Metric temperature scale for weather reports, to temperature is measured in degrees Celsius. You can use these two formulas to convert between the two:

$$
\text { degrees } F=\frac{9}{5} C+32 \quad \text { degrees } C=\frac{5}{9}(F-32)
$$

a. Water freezes at $32^{\circ}$ Fahrenheit. What is this in Celsius?
b. Water boils at $212^{\circ}$ Fahrenheit. What is this in Celsius?
c. Normal human body temperature is $98.6^{\circ}$ Fahrenheit. What is this in Celsius?
d. In sub-freezing temperatures, the wings of a plane must be deiced. The deicing chemical glycol works like antifreeze in a car, reducing the freezing point of water to as low as -50 degrees Celsius. What is this temperature in Fahrenheit?
e. The hottest day ever in Missouri reached 47.8 degrees Celsius. What is this in Fahrenheit?
f. The coldest day ever in Missouri reached -40 degrees Fahrenheit. What is this in Celsius? WHAT DO YOU NOTICE???
16. Distance Formula

Use the distance formula $(\mathbf{d}=\mathbf{r t})$ to solve the problems below.

The Shinkansen Bullet Train in Japan travels at a rate of up to 200 miles per hour.
a. At this rate, how long would it take to travel 800 miles?
b. At this rate, how long would it take to travel 1100 miles?
c. At this rate, how far could you travel in 8 hours?
d. If we had a similar train, how long would it take to travel the 300 miles from St. Louis to Chicago?
e. In a car traveling 60 miles per hour, how long does it take to travel the 300 miles from St. Louis to Chicago?
f. A car leaves St. Louis and heads for Chicago at 55 miles per hour. Another car leaves Chicago and heads for St. Louis at 45 miles per hour. How long will it take them to meet? (Remember, it's about 300 miles between the two cities.)
g. Two trains leave the same station at noon, but one goes north and the other goes south. One train is traveling twice as fast as the other. By 5:00 pm, the trains are 450 miles apart. How fast are the trains going?
h. Solve the formula $\mathrm{D}=\mathrm{rt}$ for time (that is, get $t$ by itself on one side of the equation).

## 17. Simple Interest Formula

Solve each problem, using the formula:
$\mathbf{I}=\mathbf{P R T}$, where $\mathrm{I}=$ the amount of interest in dollars
$\mathrm{P}=$ the principal in dollars
$\mathrm{R}=$ the interest rate written as a decimal
$\mathrm{T}=$ time in years
a. Solve the formula above for $P$ (that means get the variable $P$ by itself on one side of the equation).
b. If you borrow $\$ 5000$ for training expenses at $3 \%$ simple interest for 2 years, how much money will you pay in interest?
c. You invested $\$ 7000$ at $3.5 \%$ simple interest. How long will it take you to earn $\$ 1225$ in interest?
d. You invested some money at $3.25 \%$ simple interest for 5 years and earned $\$ 1218.75$ in interest. How much did you invest?
e. You invested your inheritance in two funds. You put some money in an account that pays $5 \%$ simple interest, and twice as much money in account paying 7\% simple interest. After 2 years you earned \$ 570. How much was your inheritance?

## IV. RATIOS AND Proportions

18. Gear ratio is the number of teeth each gear represents when two gears are used in a machine. For example, a pinion gear has 8 teeth and a spur gear has 28 teeth. The gear ratio is $8: 28$, which simplifies to $1: 7$. Simplify ratio below:
a. $40: 4$
b. $55: 11$ $\qquad$
c. $168: 14$ $\qquad$
d. 52:13 $\qquad$
e. $15: 10$ $\qquad$
f. $1 / 2: 1 / 3$ $\qquad$
g. $1 / 2: 5$ $\qquad$
h. $1 / 2: 3 / 4$ $\qquad$
19. Which car below gets the highest MPG, or miles per gallon?


Honda Civic
Drove 224 miles on 7 gallons


Toyota Corolla
Drove 335 miles on 15 gallons


Ford Fiesta Drove 620 miles on 20 gallons
20. 1 mole of carbon has a mass of 12 grams. What is the mass of 5.5 moles of carbon?
21. It is estimated that a 40-milliliter pond water sample contains 10,700 paramecia. How many paramecia would be found in 2000 milliliters of this water?
22. The ratio of male to female elephants in a herd is usually about 1:6. There are 300 elephants in the herd. About how many would you expect to be female?
23. 1 gram of Agarose is required to make 60 milliliters of Agarose Gel solution. How much Agarose is needed to make 865 milliliters?
24. In 1 kilogram of river sediment, there are about 156,000 insect larvae. How many larvae would be in .07 kilograms of this sediment?
25. Two tablets of ulcer medication contain 250 milligrams of medication. How many milligrams are in eight tablets?
26. The scale on a map is a ratio of actual distance to distance shown on the map. On a particular map, 1 inch $=50$ miles.
a. The distance from Point A to Point B on the map is 3 inches. What is the actual distance? Could you do this in your head?
b. The distance from Point B to Point C on the map is .5 inches. What is the actual distance? Could you do this in your head?
c. The actual distance across town is 22 miles. How will this distance appear on the map?
27. Convert the following, knowing that $\mathbf{1}$ kilogram $=\mathbf{2 . 2}$ pounds
a. A male orca can weigh up to 19,000 pounds = $\qquad$ kilograms
b. An adult tiger weighs 675 pounds $=$ $\qquad$ kilograms
c. A newborn giraffe weighs 100 kilograms = $\qquad$ pounds.

## V. Percent Problems

28. When viewing sea urchin embryonic cells undergoing mitosis, you record the following number of cells in each phase of the cell cycle. Fill in the percentages:

| Phase of Cell Cycle | Number of Cells <br> in that Phase | \% of all cells viewed that are in that <br> phase |
| :---: | :---: | :---: |
| Interphase | 54 |  |
| Prophase | 13 |  |
| Metaphase | 5 |  |
| Anaphase | 2 |  |
| Telophase | 1 |  |

29. According to www.textinganddriving.com, in 2011, nationally $23 \%$ of all auto collisions involved cell phones.
a. A total of 1.3 million collisions in the United States in 2011 involved cell phones. How many collisions were there all together?
b. 1 in 5 drivers say they have surfed the web while driving. What percent is this?
c. 10 states prohibit cell phone use while driving. What percent is this?
d. $78 \%$ of the states prohibit texting while driving. How many states is this?
30. According to a 2013 U.S. Census Bureau (www.census.gov) report, $92.1 \%$ of the 22,331 households with occupants under 35 years of age had some type of computer. How many households is this?
31. An 18 -wheeler requires $40 \%$ more time to stop than a car. If a car traveling at 60 mph takes 6.87 seconds to stop, how long would it take the 18 -wheeler to stop?
32. According to the Missouri Department of Conservation, the number of hellbender salamanders in Missouri has declined about $75 \%$ since the 1980 's, and are on course to become extinct in the next 20 years. If a particular river had 300 hellbenders in the 1980 's, how many would it have now?
33. The global human population has grown from 1 billion in the 1800 's to over 7 billion in 2012. What is the percent increase over this period?
34. The population of gray wolves in a particular area increased from 1200 to 1700 . What was the percent increase?
35. According to the Orangutan Conservancy (www.orangutan.com), the population of orangutans in Borneo and Sumatra has decreased from about 60,000 to about 40,000 in the last decade. What percent decrease is this?
36. Use the following pie graph to answer the questions below:

a. A+ Technology, Inc. made $\$ 42.85$ billion in revenue in 2014. How much of that revenue came from smartphones?
b. How much revenue came from MP3 players?
c. What percent of revenue came from services and accessories, as opposed to the products themselves?

## VI. Mixture Problems

37. How many liters of a $5 \%$-saline solution would have to be mixed with 150 liters of a $20 \%$-saline solution to create a mixture that is $10 \%$ saline?
38. A super-strength cleaning chemical must be watered down to be safe for the surface to be cleaned. How many ounces of water should be added to 8 ounces of a chemical that is $30 \%$ water to create a solution that is $80 \%$ water?
39. A wing deicer is $18 \%$ glycol. How much pure glycol and how much deicer must be added to create 100 liters of a $30 \%$-glycol deicer?
40. How many pounds of a metal containing $45 \%$ nickel must be combined with 10 pounds of a metal containing $75 \%$ nickel to create a metal that is $60 \%$ nickel?
41. Sand that cost $\$ 2.50$ per kilogram is being mixed with specially treated soil worth $\$ 7.00$ per kilogram. How much of each should be used to create 100 kilograms of a mixture worth $\$ 5.00$ per kilogram?

## VII. Function Notation

42. St. Louis Community College charges $\$ 101$ per credit hour, plus $\$ 12$ in fees. If $x=$ the number of credit hours a student takes, and $\mathrm{T}(\mathrm{x})=$ Total paid, the following function describes the situation:

$$
T(x)=101 x+12
$$

a. Calculate $\mathrm{T}(3)$, the total cost for a 3-credit course.
b. Calculate $T(15)$, the total cost to be a full-time student taking 15 credit hours.
c. Debbie paid $\$ 921$ for this semester at the Community College. How many credit hours did she take?
d. Nancy paid \$1224 for her semester. How many credit hours did she take?
e. If you have a $\$ 600$ budget, how many credit hours can you take?
43. It has been observed that the number of chirps a cricket makes per minute depends on the air temperature! Here is the function relating the temperature in degrees Fahrenheit, $t$, to the number of chirps, $\mathrm{C}(\mathrm{t})$.

$$
C(t)=4 t-160
$$

a. Calculate $\mathrm{C}(60)$, the number of chirps per minute when the air temperature is $60^{\circ} \mathrm{F}$.
b. Calculate $\mathrm{C}(80)$, the number of chirps per minute when the temperature is $80^{\circ} \mathrm{F}$.
c. You hear 40 chirps per minute. What is the temperature?
d. You hear 60 chirps per minute. What is the temperature?
e. At what temperature do you hear NO chirping from the crickets?

## Resources

Images used in section VI, question 2<br>Honda Civic 1.6 i-DTEC Elegance (IX, Facelift) by © M 93 is licensed under CC-BY-SA-3.0 (DE)<br>2014 Toyota Corolla 1.8 LE (ZRE172), front left by Mr.choppers is licensed under CC BY-SA 3.0 2009-2010 Ford Fiesta (WS) Zetec 3-door hatchback 01 is available in the public domain

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Module 2 Answer Key

### 2.3 Function Notations

1a. $f(20)=2(20)-3$

$$
=40-3
$$

$$
=37
$$

1b. $f(0)=2(0)-3$

$$
=0-3
$$

$$
=-3
$$

1c. $f(-50)$

$$
\begin{aligned}
& =2(-50)-3 \\
& =-100-3 \\
& =\mathbf{- 1 0 3}
\end{aligned}
$$

2a. $f(2)=2+7$
$=9$
2b. $f(-10)=-10+7$
$=-3$
2c. $f(8)=8+7$
$=15$
2d. $g(2)=2(2)+5$
$=4+5$
$=9$
2e. $g(-10)=2(-10)+5$

$$
\begin{aligned}
& =-20+5 \\
& =-15
\end{aligned}
$$

2f. $g(8)=2(8)+5$
$=16+5$
$=21$

2g. $h(2)=2^{2}$
$=4$
2h. $\mathrm{h}(-10)=-10^{2}$
$=100$ Check this one carefully!
2i. $h(8)=8^{2}$

$$
=64
$$

### 2.4 Career Applications: STEM

1a. 60,000
1b. 45,000
1c. 100,000
1d. 25,000
1e. 150,000
1f. $\mathbf{4 0 , 0 0 0}$
1g. 24,800
1h. 100,050
1i. $\mathbf{6 0 0 , 0 0 0}$
1j. $5 / 50,000=1 / 10,000=\mathbf{~} 0001$ liter

### 2.4 Career Applications: STEM (cont.)

2a. $x+10,000$
2b. $x-5,000$
2c. 2 x
2d. 1/2x
2e. $3 x$
2f. $x-100$
2g. $1 / 2 x-200$
2h. $2 x+50$
2i. 12x
2j. 5/x liters
3. let $\mathrm{x}=$ the number of babies $=29$

So $2 \mathrm{x}-12=$ the number of mothers $=46$
$\mathrm{x}+2 \mathrm{x}-12=75$
$3 x-12+12=75+12$
$3 x / 3=87 / 3 \quad x=29$
4. let $\mathrm{x}=$ the lighter twin $=31 \mathrm{lbs}$.

So $\mathrm{x}+1$ = the heavier twin = 32 lbs .
$\mathbf{x}+\mathbf{x}+\mathbf{1}=\mathbf{6 3}$
$2 \mathrm{x}+1-\mathbf{1}=63-1$
$2 x / 2=62 / 2 \quad x=31$
5. let $\mathrm{x}=$ the lighter twin $=\mathbf{7 8} \mathbf{l b s}$.

So $\mathrm{x}+2=$ the heavier twin $=\mathbf{8 0} \mathbf{l b s}$.
$\mathrm{x}+\mathrm{x}+\mathbf{2}=\mathbf{1 5 8}$
$2 \mathrm{x}+2-2=158-2$
$2 \mathrm{x} / 2=156 / 2 \quad \mathrm{x}=7$
6. let $\mathrm{x}=$ the lightest triplet $=37 \mathrm{lbs}$.
so $\mathrm{x}+2$ = the middle triplet $=39 \mathrm{lbs}$.
and $\mathrm{x}+4=$ the heaviest triplet $=41 \mathrm{lbs}$.
$\mathrm{x}+\mathrm{x}+2+\mathrm{x}+4=117$
$3 x+6-6=117-6$
$3 x / 3=111 / 3 \quad x=37$
7. $\quad$ Perimeter $=$ sum of all 4 sides
let $\mathrm{x}=$ the width $\quad=\mathbf{1 0} \mathbf{f t}$.
so $\mathrm{x}+2=$ the length $=\mathbf{1 2} \mathbf{f t}$.
$x+x+x+2+x+2$
$4 x+4-4=44-4$
$4 \mathrm{x} / 4=40 / 4 \quad x=10$
8. let $\mathrm{x}=$ the number of chemists $=48$
so $2 \mathrm{x}-12=$ the number of biologists $=\mathbf{8 4}$
$\mathrm{x}+2 \mathrm{x}-12=132$
$3 \mathrm{x}-12+12=132+12$
$3 x / 3=144 / 3 \quad x=48$

### 2.4 Career Applications: STEM (cont.)

9. let $\mathrm{x}=$ the length of shorter study $=\mathbf{9 5}$ days
so $\mathrm{x}+5=$ the length of the longer study $=\mathbf{1 0 0}$ days
$x+x+5=195$
$2 \mathrm{x}+5-5=195-5$
$2 \mathrm{x} / \mathbf{2}=190 / \mathbf{2} \quad \mathrm{x}=\mathbf{9 5}$
10. let $\mathrm{x}=$ the shortest route $=\mathbf{3 7}$ miles
so $\mathrm{x}+2$ = the middle route $\quad=39$ miles
and $\mathrm{x}+4=$ the longest route $=41$ miles
$\mathrm{x}+\mathrm{x}+2+\mathrm{x}+4=117$
$3 \mathrm{x}+6-6=117-6$
$3 x / 3=111 / 3 \quad x=37$
11. let $\mathrm{x}=$ the second plant

$$
\begin{aligned}
& =27 \mathrm{~cm} \\
& =24 \mathrm{~cm} \\
& =32 \mathrm{~cm}
\end{aligned}
$$

$\mathrm{x}+\mathrm{x}-3+\mathrm{x}+5=83$
$3 x+2-2=83-2$
$3 x / 3=81 / 3 \quad x=27$
12. let $\mathrm{x}=2013=49$
so $\mathrm{x}-20=2012=29$
and $\mathrm{x}-2=2014=47$
$\mathrm{x}+\mathrm{x}-\mathbf{2 0}+\mathrm{x}-2=125$
$3 x-22+22=125+22$
$3 x / 3=147 / 3 \quad x=49$
13. let $\mathrm{x}=$ cost of iPad $=\$ 530$
so $2 \mathrm{x}+290=$ MacBook $=\mathbf{\$ 1 3 5 0}$
$\mathrm{x}+2 \mathrm{x}+290=1880$
$3 \mathrm{x}+290-290=1880-290$
$3 \mathrm{x} / \mathbf{3}=1590 / 3 \quad \mathbf{x}=530$
14a. about $38,079.6$ Joules
14b. $V^{2}=324$, so $V=$ the square root of 324
so 18 meters/sec
14c. about $34,143.4$ kilograms
14d. about 980.3 meters/sec
14e. about 147.2 meters/sec
14f. $\mathrm{m}=\frac{2 K E}{V^{2}}$

15a. $0^{\circ} \mathrm{C}$
15b. $100^{\circ} \mathrm{C}$
15c. $37^{\circ} \mathrm{C}$
15d. $F=9 / 5(-50)+32=-58$ degrees $F$
15e. $F=9 / 5(47.8)+32=\mathbf{1 1 8 . 0 4}$ degrees $F$
15f. -40 degrees - they are the SAME!
16a. 4 hours
16b. $1100=200$ t, so $\mathbf{t}=5.5$ hours
16c. 1600 miles
16d. 1.5 hours
16e. 5 hours

16f. combined rate $=100 \mathrm{mph}$, so about 3 hours
$\mathbf{1 6 g}$. $\mathrm{r}=$ slower train $=\mathbf{3 0} \mathbf{m p h}$
$2 \mathrm{r}=$ faster train $=\mathbf{6 0} \mathbf{m p h}$
$450=3 r(5) \quad 450=15 r \quad r=30$
16h. $t=D / r$
17a. $P=\frac{I}{R T}$
17b. $\mathrm{I}=5000(.03) 2$
$\mathrm{I}=\mathbf{\$ 3 0 0}$
17c. $1225=7000(.035) t$
$1225=245 \mathrm{t}$
$\mathrm{t}=\mathbf{\$ 3 0 0}$
17d. $1218.75=\mathrm{P}(.0325)(5)$
$1218.75=.1625 \mathrm{P}$
$\mathrm{t}=\mathbf{\$ 7 5 0 0}$
17e. let $x=$ the amount you invested in $5 \%$ fund
So $2 \mathrm{x}=$ the amount you invested in $7 \%$ fund
$570=.05(2)(\mathrm{x})+.07(2)(2 \mathrm{x})$
$570=.1 \mathrm{x}+.28 \mathrm{x}=.38 \mathrm{x}$
$\mathrm{x}=\mathbf{\$ 1 5 0 0}$ in 5\% fund
so $2 \mathrm{x}=\$ 3000$ in $7 \%$ fund, and total $\$ 4500$
18a. 10:1
18b. 5:1
18c. 12:1
18d. 4:1
18e. 3:2
18f. $\frac{1}{2} \div \frac{1}{3}=\frac{1}{2} \cdot \frac{3}{1}=\frac{3}{2}=3: 2$
18g. $\frac{1}{2} \div \frac{5}{1}=\frac{1}{2} \cdot \frac{1}{5}=\frac{1}{10}=1: 10$
18h. $\frac{1}{2} \div \frac{3}{4}=\frac{1}{2} \cdot \frac{4}{3}=\frac{2}{3}=2: 3$
19. Honda $=224 / 7=32 \mathrm{mpg}$ is highest

Toyota $=335 / 15=22.3 \mathrm{mpg}$
Ford $=620 / 20=31 \mathrm{mpg}$
20. $\frac{1 \mathrm{~mole}}{12 \text { grams }}=\frac{5.5}{\mathrm{x} \text { grams }}$
$\mathrm{x}=5.5(12)=\mathbf{6 6}$ grams
21. $\frac{40 \mathrm{ml}}{10,700 \text { paramecia }}=\frac{2000 \mathrm{ml}}{\mathrm{x} \text { paramecia }}$
$40 x=21,400,000$
$\mathrm{x}=535,000$ paramecia
22. $\frac{6 \text { female }}{7 \text { total elephants }}=\frac{\mathrm{x} \text { females }}{300 \text { total elephants }}$
$7 \mathrm{x}=1800$
$\mathrm{x}=$ about 257 female elephants

### 2.4 Career Applications: STEM (cont.)

23. $\frac{1 \mathrm{gram}}{60 \mathrm{ml}}=\frac{\mathrm{x} \text { grams }}{865 \mathrm{ml}}$
$60 \mathrm{x}=865$
$\mathrm{x}=$ about $\mathbf{1 4 . 4}$ grams Agarose
24. $\frac{1 \mathrm{~kg}}{156,000 \text { larvae }}=\frac{.07 \mathrm{~kg}}{\mathrm{x} \text { larvae }}$
$\mathrm{x}=\mathbf{1 0 , 9 2 0}$ larvae
25. $\frac{2 \text { tablets }}{250 \mathrm{mg}}=\frac{8 \text { tablets }}{\mathrm{x} \mathrm{mg}}$
$2 \mathrm{x}=2000, \mathrm{x}=1000 \mathrm{mg}$
26a. $\frac{1 \text { inch }}{50 \text { miles }}=\frac{3 \text { inches }}{x \text { miles }}$
$\mathrm{x}=150$ miles
26b. $\frac{1 \text { inch }}{50 \text { miles }}=\frac{.5 \text { inches }}{x \text { miles }}$
$\mathrm{x}=25$ miles
26c. $\frac{1 \text { inch }}{50 \text { miles }}=\frac{x \text { inches }}{22 \text { miles }}$
$50 \mathrm{x}=22, \mathrm{x}=$ about .4 inches
27a. $\frac{1 \mathrm{~kg}}{2.2 \mathrm{lbs}}=\frac{\mathrm{x} \mathrm{kg}}{19,000 \mathrm{lbs}}$
$2.2 \mathrm{x}=19000, \mathrm{x}=8636.4$ kilograms
27b. $\frac{1 \mathrm{~kg}}{2.2 \mathrm{lbs}}=\frac{\mathrm{xkg}}{675 \mathrm{lbs}}$
$2.2 x=675, x=306.8$ kilograms
27c. $\frac{1 \mathrm{~kg}}{2.2 \mathrm{lbs}}=\frac{100 \mathrm{~kg}}{\mathrm{x} \text { lbs }}$
$x=220$ pounds
26. 

| Phase | \# Cells | \% of All Cells |
| :---: | :---: | :---: |
| Interphase | 54 | $54 / 75=\mathbf{7 2 \%}$ |
| Prophase | 13 | $13 / 75=\mathbf{1 7 . 3} \%$ |
| Metaphase | 5 | $5 / 75=\mathbf{6 . 7} \%$ |
| Anaphase | 2 | $2 / 75=\mathbf{2 . 7 \%}$ |
| Telophase | 1 | $1 / 75=\mathbf{1 . 3} \%$ |

29a. $1.3=23 \%$ of the total accidents
$1.3=.23 \mathrm{x}$
$\mathrm{x}=$ about 5.65 million accidents
29b. $1 / 5=20 \%$
29c. $10 / 50=1 / 5=20 \%$
29d. $.78(50)=39$ states
30. . $921(22,331)=$ about 20,567
31. $6.87+.4(6.87)=$ about 9.62 seconds
32. $300-.75(300)=75$ hellbenders left
33. Amount of increase $=6$ billion.
$6=x(1)$
$x=6$, so $\mathbf{6 0 0 \%}$ increase
34. Amount of increase $=500$

500 is what percent of 1200 ?
$500=x(1200)$
$\mathrm{x}=500 / 1200=.4166$
= about 42\% increase
35. Amount of decrease $=20,000$

20,000 is what percent of 60,000 ?
$20000=x(60000)$
$\mathrm{x}=20000 / 60000=.3333$
= about 33\% decrease
36a. $42 \%$ of 42.85 billion

$$
=.42(42.85)=\text { almost } \$ 18 \text { billion }
$$

36b. $1 \%$ of 42.85 billion
$=.01(42,850,000,000)=\$ 428,500,000$
36c. $8 \%+4 \%=12 \%$
37. let $\mathrm{x}=$ the number liters of $5 \%$ solution
final solution will have $150+x$ liters
$.05 \mathrm{x}+.2(150)=.1(150+\mathrm{x})$
$.05 \mathrm{x}+30=15+.1 \mathrm{x}$
$15=.05 x$
$\mathrm{x}=15 / .05$
$x=300$ liters of $5 \%$ saline solution
38. let $\mathrm{x}=$ the number of ounces of water final solution will have $\mathrm{x}+8 \mathrm{oz}$
Remember that pure water contains $100 \%$ water, which = 1
$.3(8)+1 x=.8(x+8)$
$2.4+x=.8 x+6.4$
. $2 \mathrm{x}=4$
$x=20$ ounces of water
39. let $\mathrm{x}=$ the number of liters of pure $(100 \%)$ glycol

We need 100 liters total, so we'll need ( $100-\mathrm{x}$ ) liters of deicer
$1 \mathrm{x}+.18(100-\mathrm{x})=.3(100)$
$\mathrm{x}+18-.18 \mathrm{x}=30$
$.82 \mathrm{x}=12$
$\mathrm{x}=12 / .82=14.63$
$x=$ about 14.5 liters of glycol
$100-\mathrm{x}=$ about 85.5 liters of deicer
40. let $\mathrm{x}=$ the number of pounds of $45 \%$ nickel
$10+\mathrm{x}=$ the number of pounds in the alloy being created
$.45 \mathrm{x}+.75(10)=.6(10+\mathrm{x})$
$.45 \mathrm{x}+7.5=6+.6 \mathrm{x}$
$1.5=.15 x$
$x=1.5 / .15=10$
$x=10 \mathrm{lbs}$. of $45 \%$ nickel
This makes sense, because using equal amounts of each material will create a percent nickel that is halfway between $45 \%$ and $75 \%$

### 2.4 Career Applications: STEM (cont.)

41a. let $\mathrm{x}=$ the number of kilograms of sand
$100-\mathrm{x}=$ the number of kilograms of soil
$2.5 x+7(100-x)=5(100)$
$2.5 \mathrm{x}+700-7 \mathrm{x}=500$
$-4.5 \mathrm{x}+700=500$
$200=4.5 \mathrm{x}$
$x=200 / 4.5=44.44$
$x=$ about 44 kilograms of sand
$100-x=$ about 56 kilograms of soil
42a. $T(3)=101(3)+12$
$T(3)=303+12=\$ 315$
42b. $T(15)=101(15)+12$
$\mathrm{T}(15)=1515+12=\$ 1527$
43a. $C(60)=4(60)-160$
$C(60)=240-160=\mathbf{8 0}$ chirps per minute
43b. $C(80)=4(80)-160$
$C(80)=320-160=160$ chirps per minute
43c. $40=4 \mathrm{t}-160$
$4 \mathrm{t}=40+160$
$t=50$ degrees Fahrenheit
43c. $60=4 \mathrm{t}-160$
$4 \mathrm{t}=60+160$
t = 55 degrees Fahrenheit
43c. $0=4 \mathrm{t}-160$
$4 \mathrm{t}=0+160$
t = 40 degrees Fahrenheit
42c. $921=101(\mathrm{x})+12$
$921-12=101 x$
$101 \mathrm{x}=909$
$x=9$ credit hours
42d. $1224=101(\mathrm{x})+12$
$1224-12=101 \mathrm{x}$
$101 \mathrm{x}=1212$
$\mathrm{x}=12$ credit hours
42d. $600=101(\mathrm{x})+12$
$600-12=101 x$
$101 \mathrm{x}=588$
$x=5.88$ so 5 credit hours

Adult Learning Academy<br>Elementary Algebra Workbook

## Module 3: The Rectangular Coordinate System, Graphs of Linear equations, Slope

Learning Objectives
By the time you finish this module, you should be able to:Identify the x-axis, y-axis, origin, and Quadrants I, II, III, and IV on a coordinate grid.Plot any point on the coordinate grid, including points on the axesGiven the equation of any line, graph it by making a table.Recognize and graph the equation of any horizontal line and any vertical line.Recognize when a given equation is NOT linear, and graph it by making a table.Find the $x$-intercept and the $y$-intercept of any linear equation.Find the slope of any line by looking at the graph.Find the slope of any line if you know two points.Identify the slope of horizontal and vertical lines.Graph a line when you are given a point and a slope.
Graph a line when you are given its equation in Slope-Intercept ( $y=m x+b$ ) form.
$\square$ Write any linear equation in Slope-Intercept $(y=m x+b)$ form.

## IMPORTANT INFORMATION FROM MODULE 3:

The $x$-axis is the horizontal axis, and the independent variable. The $y$-axis is the vertical axis, and the dependent variable. Every point has an x-coordinate and a y-coordinate written as an ordered pair ( $\mathrm{x}, \mathrm{y}$ ). The origin has coordinates $(0,0)$.

Points on the $x$-axis are called $x$-intercepts, and their $y$-coordinate is $0 .(3,0),(2,0)$, etc.
To find the x -intercept, plug 0 in for y and find the x -value that goes with it.
Points on the $y$-axis are called $y$-intercepts, and their $x$-coordinate is $0 .(0,3),(0,2)$, etc. To find the $y$-intercept, plug 0 in for x and find the y -value that goes with it.

The equation $\mathrm{y}=\mathrm{a}$ number is always horizontal. The slope of a horizontal line is 0 . The equation $\mathrm{x}=$ a number is always vertical. The slope of a vertical line is undefined.

SLOPE: Given any two points $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$, the slope of the line containing them is $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$. Slope is also $\frac{\text { rise }}{\text { run }}$, or the rate of change.

St. Louis Community College

| Topic | Website | Videos | Exercises |
| :---: | :---: | :---: | :---: |
| Plotting Points | www.khanacademy.org | Descartes \& Cartesian Coordinates | Graphing Points |
|  |  | The Coordinate Plane | Graphing Pts \& Naming Quad. |
|  |  | Plot Ordered Pairs | Points on the Coordinate Plane |
|  |  | Quadrants of Coordinate Plane |  |
| Points \& Equations | www.khanacademy.org | Ordered Pair Solutions of Eq 2 | Ordered Pair Sol's to Linear Eq. |
|  |  | Plotting (x,y) Relationships | Identifying Linear Relationships |
|  |  | Graphs of Linear Equations |  |
|  |  | Application Problem with Graph |  |
|  |  | Interpreting Linear Graphs |  |
|  |  | Exploring Linear Relationships |  |
|  |  | Recognizing Linear Functions |  |
|  |  | Graphing Lines 1 |  |
| Intercepts | www.khanacademy.org | Graphing Using x and y Intercepts | Solving for the X-Intercept |
|  |  | Graphing Using Intercepts |  |
|  |  | X and Y Intercepts |  |
|  |  | $X$ and $Y$ Intercepts 2 |  |
| Horizontal \& Vertical | http://www.youtube.com/watch?v=KwBE2pJDWvU | Horizontal and Vertical Lines |  |
|  | http://www.youtube.com/watch?v=VMitkRc5jHA | Horizontal and Vertical Lines |  |
| Slope | www.khanacademy.org | Slope of a Line | Identifying Slope of a Line |
|  |  | Slope of a Line 2 | Line Graph Intuition |
|  |  | Slope and Rate of Change |  |
|  |  | Graphical Slope of a Line |  |
|  |  | Slope of a Line 3 |  |
|  |  | Slope and y-intercept Intuition |  |
| Slope-Intercept Form | www.khanacademy.org | Graph Line in Slope-Intercept Form | Graphing Linear Equations |
|  |  | Converting to Slope-Intercept Form | Slope-Intercept Form |
|  |  | Linear Eq in Slope-Intercept Form |  |


| Topic | Website | Videos | Exercises |
| :--- | :--- | :--- | :--- |
| Slope-Intercept Form | www.khanacademy.org | Graphs Using Slope-Intercept Form |  |
| (Cont.) |  | Equation of a Line 1 |  |
|  |  | Equation of a Line 2 |  |
| Module 3 Test Review | www.stlcc.edu | Blackboard PowerPoint | Ten-Second Graphing |
|  |  | Blackboard PowerPoint | Ten-Second Graphing Version 2 |
|  |  | Blackboard PowerPoint | Module 3 Review Flashcards |

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Adult Learning Academy Elementary Algebra Workbook
3.1 Point-Plotting Practice

There are 4 parts to this picture.

1. Plot these points and connect them in order:
$(6,8)(5,6)(6,4)(2,-4)(0,-5)$
$(-1,-6)(-2,-9)(-1,-11)(0,-12)$
$(0,-13)(-3,-12)(-3,-11)(-4,-9)$
$(-4,-6)(-6,-9)(-8,-11)(-8,-12)$
$(-1,-14)(-7,-14)(-10,-13)$
$(-10,-11)(-8,-9)(-6,-3)(-5,1)$
$(-3,3)$
$(2,6)(2,10)(3,11)(5,12)$
$(9,10)(9,9)(6,9)(9,8)(8,7)$
$(6,8)$
2. Plot these points and connect them in order:
$(5.5,3)(7,3)(7,2)(5,2)$
3. Plot these points and connect them in order:
$(3,3)(5,1)(7,1)(7,0)(4,0)$
$(2,2)$
4. Plot these points and connect them in order:
$(2,-4)(3,-11)(4,-12)(4,-13)$
$(1,-12)(1,-11)(0,-5)$

What do you see?
Plot the points on the left hand side to find out!


## Adult Learning Academy Elementary Algebra Workbook <br> 3.2 GRAPHING PrActice

Make a table and plot points to graph the equations.

1. $2 \mathrm{x}-\mathrm{y}=1$
2. $y=1 / 2 x-3$




3. $y=-3 x$
4. $y=x^{2}$



5. $\mathrm{y}=\mathrm{x}$
6. $x+y=5$





Make a table and plot points to graph the equations.
7. $\mathrm{x}-\mathrm{y}=1$
8. $y=-2 x+3$


9. $y=3 / 4 x$

11. $y=|x|$

12. $\mathrm{y}=-2 \mathrm{x}+3$



10. $\mathrm{y}=\mathrm{x}-3$



## Special Lines: Horizontal, Vertical

Question: Where does a 3,000-pound rhinoceros sleep?
Answer: Anywhere it wants to!

13. $y=3$



A Horizontal line ALWAYS has the equation $\mathrm{y}=\mathrm{a}$ number
14. $x=5$



A Vertical line ALWAYS has the equation $\mathrm{x}=\mathrm{a}$ number

Graph each equation using any method
15. $\mathrm{y}=1 / 4 \mathrm{x}-2$

18. $\mathrm{y}=7$

21. $y=-3 / 4 x+2$

16. $3 x$

19. $y=-2 / 3 x+1$

22. $\mathrm{y}-\mathrm{x}=3$

17. $1 / 2 \mathrm{x}+3$

20. $x=4$
23. $3 x+5 y=15$


Graph each equation using any method
24. $y=-2 x+1$

27. $x=-4$

30. $y=-\frac{4}{3} x$

25. $\mathrm{y}=\frac{3}{5} \mathrm{x}$

28. $y=\frac{1}{5} x+3$

31. $\mathrm{x}+\mathrm{y}=5$

26. $y=1 / 2 x-3$

29. $y=-2$

32. $\mathrm{y}=\mathrm{x}^{2}-3$


Graph the line through the given point, with the given slope
33. Point (3, 1) Slope $1 / 2$

36. Point (-3, -2) Slope 4

39. Point $(2,-1)$ Slope $-\frac{3}{2}$

34. Point $(-4,1)$ Slope $-\frac{2}{3}$

37. Point $(-4,0)$ Slope -2

40. Point (4, 0) Slope undefined

35. Point $(0,0)$ Slope $\frac{1}{3}$

38. Point $(-2,3)$ Slope 0

41. Point $(-1,0)$ Slope 1

Adult Learning Academy Elementary Algebra Workbook
3.3 Slope Practice

1. Slope practice: What is the slope of each line below?
a.

b.

d.

e.

c.

f.

2. Calculate the slope of the line containing each pair of points. Graph each line.


c. $(3,-1)$ and $(-4,5)$

d. $(3,-1)$ and $(5,2)$
f. $(3,6)$ and $(7,6)$
h. ( $-3,4$ ) and $(1,-2)$
3. If a line has positive slope, then its graph $\qquad$ from left to right.
4. If a line has negative slope, then its graph $\qquad$ from left to right.

# Adult Learning Academy 

## Elementary Algebra Workbook

3.4 Practice with Slope-Intercept Form

## Slope-Intercept Form y=mx+b

Rewrite each equation in slope-intercept form and graph the line.



1. Carefully graph each to create a design below:
a. $y=\frac{3}{2} x+8$
b. The line through the point $(-6,1)$ with slope $\frac{-3}{2}$
c. $\mathrm{y}=4$
d. The line through the point $(2,-4)$ with a slope of 0 .
e. $y=\frac{3}{2} x-8$
f. $3 x+2 y=16$


## Elementary Algebra Workbook

3.6 CAREER APPLICATIONS: STEM

Remember function notation from Module 2? Believe it or not, $f(x)$ is just another way of saying " $y$ "! Any equation that starts with $y=m x+b$ could also be written as $f(x)=m x+b$. The advantage of the $f(x)$ notation is that you can tell exactly what number to substitute in for $x$ !

1. Forensic scientists can approximate a person's height by measuring the person's femur, which is the long bone that stretches from the hip socket to the kneecap. The relationship can be expressed as a function:

For females:
$f(x)=2.3 x+61 \quad$ where $x$ is the length of the femur in $c m$, and $f(x)$ is the woman's height in centimeters.
For males:
$m(x)=2.2 x+69 \quad$ where $x$ is the length of the femur in $c m$, and $m(x)$ is the man's height in centimeters
a. Fill in the tables:

| WOMAN |  |
| :---: | :---: |
| Length of femur, <br> $X$ | Height in cm, <br> $\mathrm{f}(\mathrm{x})$ |
| 40 cm |  |
| 45 cm |  |
| 50 cm |  |
| 55 cm |  |


| MAN |  |
| :---: | :---: |
| Length of femur, <br> $X$ | Height in cm, <br> $\mathrm{f}(\mathrm{x})$ |
| 40 cm |  |
| 45 cm |  |
| 50 cm |  |
| 55 cm |  |

b. Graph it!

Height in cm

c. What is the slope and the f-intercept for the women's equation?
d. What is the slope and they m-intercept for the men's equation?
2. The dose for a particular medication is 5 milliliters per kilogram that the patient weighs. This relationship can be expressed as the function $d(x)=5 x$, where $d$ is the dose in milliliters and $x$ is the weight of the patient in kilograms.
a. Calculate the dose for patients whose weights are listed:

| Weight in kilograms, <br> x | Dose in milliliters, <br> $\mathrm{d}(\mathrm{x})$ |
| :---: | :---: |
| Baby, 5 kg |  |
| Child, 20 kg |  |
| Woman, 50 kg |  |
| Man, 60 kg |  |

b. What is the slope of this line? What is the d-intercept?
c. What does the slope mean in this problem? Why is it positive?
d. If we graphed this line, we would only show points in the first quadrant. Why?
3. In Module 2, you worked with the formula for converting Celcius temperature to Fahrenheit. Now you can see that this formula is a linear equation!

$$
F=\frac{9}{5} C+32
$$

You can also write the fraction as a decimal, so the formula becomes

$$
\mathrm{F}=1.8 \mathrm{C}+32
$$

a. What is the slope of the equation? What is its F-intercept?
b. Fill in the table:

| $\mathrm{C}^{\circ}$ | $\mathrm{F}^{\circ}$ |
| :---: | :---: |
| 0 |  |
| 10 |  |
| 20 |  |
| 30 |  |

c. Sketch the graph:

4. Remember the problem from Module 2 about how long it costs to take courses at St. Louis Community College? Now you can recognize that function as a linear equation!

St. Louis Community College charges $\$ 101$ per credit hour, plus $\$ 12$ in fees. If $x=$ the number of credit hours a student takes, and $y=$ total paid, the following function describes the situation:

$$
y=101 x+12
$$

a. What is the $y$ intercept of this line? What does it tell you?
b. What is the slope of the line? Why is it positive?
c. The line for this problem should only be graphed in the first quadrant. Why?
d. Graph it! Choose an appropriate scale for the $x$-axis and the $y$-axis.

5. Remember the cricket problem from Module 2? You can now recognize the function as a linear equation!

It has been observed that the number of chirps a cricket makes per minute depends on the air temperature! Here is the function relating the temperature in degrees Fahrenheit, $x$, to the number of chirps, $y$.

$$
y=4 x-160
$$

a. What is the slope of this line? Why is it positive?
b. What is the x-intercept of the line? What does it tell you?
c. Why does this graph make the most sense in the first quadrant?
d. Graph it! Choose an appropriate scale for the x -axis and the y -axis.


Adult Learning Academy Elementary Algebra Workbook<br>Module 3 Answer Key

### 3.1 Point-Plotting Practice

A tyrannosaurus rex

3.2 Graphing Practice
1.

5.

9.

13.

17.

2.

3.

6.

10.

14.

18.

7.

11.

15.

19.

4.

8.

12.

16.

20.


### 3.2 Graphing Practice (cont.)

21. 


25.

29.

33.

37.

41.

22.

26.

30.

31.

34.

35.

39.

27.

24.

28.

32.

36.

40.


### 3.3 Slope Practice

1a. $-1 / 3$
1b. $2 / 3$
1c. $1 / 2$
1d. $-1 / 2$
1e. 0
1f. Undefined
Graphs are approximate!
2a. $m=\frac{3-0}{5-0}=3 / 5$

2b. $m=\frac{-3-0}{5-0}=-3 / 5$


2c. $m=\frac{5-(-1)}{-4-3}=-6 / 7$

2d. $m=\frac{2-(-1)}{5-3}=3 / 2$


2e. $m=\frac{-4-(-2)}{-2-4}=\frac{-2}{-6}=1 / 3$


2f. $m=\frac{6-6}{7-3}=\frac{0}{4}=0$ (horizontal)

2g. $m=\frac{7-6}{3-3}=\frac{1}{0}=$ Undefined (vertical)

2h. $m=\frac{-2-4}{1-(-3)}=\frac{-6}{4}=-3 / 2$

3. goes uphill
4. goes downhill
3.4 Slope-Intercept Form

|  | SIFORM | SLOPE | Y-INT. | GRAPH |
| :---: | :---: | :---: | :---: | :---: |
| A. | $y=-3 / 2 x+3$ | $\mathbf{m}=-3 / 2$ | $(0,3)$ |  |
|  |  |  |  | 1 |
| B. | $y=2 x+4$ | $\mathrm{m}=2$ | $(0,4)$ | , |
|  |  |  |  |  |
| C. | $y=-1 / 2 x+3 / 2$ | $\mathrm{m}=-1 / 2$ | (0, 3/2) | $\cdots$ |
|  |  |  |  |  |
|  |  |  |  |  |
| D. | $y=1 / 2 x-5 / 2$ | $\mathrm{m}=1 / 2$ | ( $0,-\mathbf{5} / 2$ ) | \% |
|  |  |  |  | , |
|  |  |  |  | 1 |
| E. | $\mathbf{y}=3 \mathbf{x}$ | $\mathbf{m}=3$ | (0, 0) | - |
|  |  |  |  |  |
|  |  |  |  |  |
| F. | $\mathbf{y}=-3 \mathrm{x}$ | $\mathrm{m}=-3$ | (0, 0) |  |
|  |  |  |  |  |
|  |  |  |  | $\bigcirc$ |
| G. | $y=1 / 2 x+7$ | $\mathrm{m}=1 / 2$ | $(0,7)$ |  |
|  |  |  |  |  |
| H. |  |  |  | \# $\quad 1$ |
| H. | $y=x-2$ | $\mathrm{m}=1$ | (0, -2) | \% |
|  |  |  |  | \| |


|  | SIFORM | SLOPE | Y-INT. | GRAPH |
| :---: | :---: | :---: | :---: | :---: |
| I. | $y=2 x-5$ | $\mathrm{m}=2$ | $(0,-5)$ |  |
| J. | $y=5 / 3 x-5$ | $\mathrm{m}=5 / 3$ | $(0,-5)$ |  |
| K. | $y=-3 / 4 x+5$ | $m=-3 / 4$ | $(0,5)$ | $\#$ |
| L. | $y=-2$ | $\mathbf{m}=0$ | (0, -2) |  |
| M. | $x=-5$ | undefined | n/a |  |
| N. | $\mathrm{y}=1$ | $\mathbf{m}=0$ | $(0,1)$ |  |
| 0 | You'll learn to create the equation for this line in the next module! | $\mathrm{m}=2 / 3$ | Estimate if necessary | $\ldots$ |
| P. | You'll learn to create the equation for this line in the next module! | $\mathrm{m}=-1$ | Estimate if necessary | $\because$ |

### 3.5 Line Designs!

1. Star

2. Tee-pee


### 3.6 Career Applications - STEM

11. 

| WOMAN |  |
| :---: | :---: |
| Length of $F$ <br> $X$ | Height in cm, <br> $\mathrm{f}(\mathrm{x})$ |
| 40 cm | $\mathbf{1 5 3 ~ c m}$ |
| 45 cm | $\mathbf{1 6 4 . 5} \mathbf{~ c m}$ |
| 50 cm | $\mathbf{1 7 6} \mathrm{~cm}$ |
| 55 cm | $\mathbf{1 8 7 . 5} \mathrm{~cm}$ |


| MAN |  |
| :---: | :---: |
| Length of $F$ <br> X | Height in cm, <br> $\mathrm{f}(\mathrm{x})$ |
| 40 cm | $\mathbf{1 5 7} \mathbf{~ c m}$ |
| 45 cm | $\mathbf{1 6 8} \mathbf{~ c m}$ |
| 50 cm | $\mathbf{1 7 9} \mathbf{~ c m}$ |
| 55 cm | $\mathbf{1 9 0} \mathbf{~ c m}$ |

1b.


1c. $\mathbf{m}=2.3, \mathrm{f}$-intercept $=61$
1d. $m=2.2, f$-intercept $=69$

### 3.6 Career Applications - STEM (cont.)

2a.

| Weight in kg <br> x | Dose in mL <br> $\mathrm{d}(\mathrm{x})$ |
| :---: | :---: |
| Baby, 5 kg | $\mathbf{d}(\mathbf{5})=\mathbf{5 ( 5 )}=\mathbf{2 5}$ |
| Child, 20 kg | $\mathrm{~d}(\mathbf{2 0})=\mathbf{5 ( 2 0 )}=\mathbf{1 0 0}$ |
| Woman, 50 kg | $\mathbf{d ( 5 0 )}=\mathbf{5 ( 5 0 )}=\mathbf{2 5 0}$ |
| Man, 60 kg | $\mathbf{d ( 6 0 )}=\mathbf{5 ( 6 0 )}=\mathbf{3 0 0}$ |

2b. $\mathbf{m}=5$, $\mathbf{d}$-intercept $=0$
2c. for every kilogram you add in weight, the dose increases by 5 milliliters; the slope is positive because as weight increases, so does size of the dose
2d. Weight and doses are non-negative quantities

3a. $\mathbf{m}=1.8$, F-intercept $=32$
3b.

| $\mathrm{C}^{\circ}$ | $\mathrm{F}^{\circ}$ |
| :---: | :---: |
| 0 | $\mathbf{3 2}$ |
| 10 | $\mathbf{5 0}$ |
| 20 | $\mathbf{6 8}$ |
| 30 | $\mathbf{8 6}$ |

3c.


4a. y-int. = 12; this tells you the fee, regardless of the number of credit hours
4b. $m=101$, this number is positive because as credit hours increase, so does the cost
4c. Both credit hours and fees are positive quantities

### 3.6 Career Applications - STEM (cont.)

4d.


5a. $\mathrm{m}=4$; slope is positive because the higher the temperature, the faster they chirp
5b. $\mathbf{x}$-intercept $=40$; this tells you that crickets stop chirping at 40 degrees
5 c . The number of chirps must be positive, and crickets aren't out in negative temperatures
5d.


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Adult Learning Academy Elementary Algebra Workbook

Module 4: Equations of lines, graphs of Linear inequalities, SOLVING SYSTEMS BY GRAPH

## Learning ObJectives

By the time you finish this module, you should be able to:Given a point and a slope, graph and write the equation of the line.
$\square$ Given two points, graph and write the equation of a line containing them.
$\square$ Write the equation of a line in standard form, in point-slope form, and in slopeintercept form.
$\square$ Graph a system of two equations and find the solution by locating their intersection.
$\square$ Recognize and graph parallel lines.
$\square$ Recognize and graph perpendicular lines.
$\square$ Find the slope of parallel and perpendicular lines.
$\square$ Graph linear inequalities by shading.

## IMPORTANT INFORMATION FROM MODULE 4:

SLOPE: Given any two points $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$, the slope of the line containing them is $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$.

SLOPE-INTERCEPT FORM OF A LINEAR EQUATION:
$y=m x+b$, where $m$ is the slope and $b$ is the y -intercept.
POINT-SLOPE FORM OF A LINEAR EQUATION:
$y-y_{1}=m\left(x-x_{1}\right)$, where $m$ is the slope and $\left(x_{1}, y_{1}\right)$ is a point on the line.
STANDARD FORM OF A LINEAR EQUATION:
$A x+B y=C$, where $A, B$, and $C$ are constants.
PARALLEL LINES have equal slopes.
PERPENDICULAR LINES have opposite, reciprocal slopes.
TO GRAPH A LINEAR INEQUALITY: 1) draw the border line solid for > or <, dotted for > or <;
2) test a point not on the line; 3) shade the "good" side of the line


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## Adult Learning Academy <br> Elementary Algebra Workbook <br> 4.1 How to Write the equation of a Line

| Standard Form: | Slope-Intercept Form: | Point-Slope Form: | Slope Formula: |
| :---: | :---: | :---: | :---: |
| $\mathbf{A x}+\mathbf{B y}=\mathbf{C}$ | $\mathbf{y}=\mathbf{m x}+\mathbf{b}$ | $\mathbf{y}-\mathbf{y 1}=\mathbf{m}(\mathbf{x}-\mathbf{x 1})$ | $\mathbf{m}=\frac{y_{2-y_{1}}}{x_{2-}-x_{1}}$ |


| If you know the SLOPE and the Y-INTERCEPT, use the SLOPE-INTERCEPT FORM | If you know the SLOPE and any old POINT, use the POINT-SLOPE FORM | If you know 2 POINTS that are on the line, use them to find the SLOPE. Then pick one of the points and use the POINT-SLOPE FORM |
| :---: | :---: | :---: |
| 1a. Write the equation of a line with a slope of $-1 / 22$ and $y$-intercept at $(0,3)$. | 2a. Write the equation of a line with a slope of $-1 / 2$ that passes through the point $(4,7)$ | 3a. Write the equation of a line that contains the points $(3,7)$ and $(-2,1)$. |
| 1b. Write the equation of a line with a slope of $3 / 4$ and $y$-intercept at $(0,-2)$. | 2 b . Write the equation of a line with a slope of $3 / 4$ that passes through the point $(-2,5)$ | 3b. Write the equation of a line that contains the points $(0,5)$ and $(2,-3)$. |

## Adult Learning Academy <br> Elementary Algebra Workbook <br> 4.2 Parallel and Perpendicular lines

1. Using a ruler, carefully graph each of these lines on the one coordinate system below. What do you notice?
a. $y=1 / 2 x+3$
b. $y=1 / 2 x-2$
c. $y=1 / 2 x$
d. $y=1 / 2 x+5$
e. $y=1 / 2 x-4$
f. $y=1 / 2 x-8$

2. PARALLEL lines never intersect (cross). Parallel lines have equal slopes. Determine if each pair of lines below is parallel.
a. $y=-3 x+1$ and $y=-3 x-4$
b. $y=3 x+5$ and $y=2 x+5$
c. $y=3 x+5$ and
d. $x+y=5$ and $x+y=10$
e. $2 x+3 y=6$ and $y=-2 / 3 x+4$
f. $x=5$
and $\quad y=5$

Are they parallel? $\qquad$

Are they parallel? $\qquad$

Are they parallel? $\qquad$

Are they parallel? $\qquad$

Are they parallel? $\qquad$

Are they parallel? $\qquad$

### 4.2 PARALLEL AND PERPENDICULAR LINES (CONT.)

3. Using a ruler, carefully graph each of these lines on the one coordinate system below. What do you notice?

$$
y=\frac{3}{4} x+3 \quad y=\frac{-4}{3} x+1
$$


4. PEPENDICULAR lines intersect (cross) at a right angle ( 90 degrees). Perpendicular lines have OPPOSITE (one positive, one negative), RECIPROCAL (upside-down fractions) slopes.

For example, if one line has a slope of $\frac{1}{3}$, a line perpendicular to it would have slope $\frac{-3}{1}$ or -3 .

For each pair of equations below, circle whether the two lines are parallel (same slope), perpendicular (opposite reciprocal slopes), or neither.
a. $y=-3 x+1$ and $y=-3 x-4 \quad$ Parallel Perpendicular Neither
b. $y=3 x+5$ and $y=2 x+5 \quad$ Parallel Perpendicular Neither
c. $y=\frac{2}{3} x+5$ and $y=\frac{-3}{2} x+2 \quad$ Parallel $\quad$ Perpendicular Neither
d. $y=4 x+1$ and $y=-4 x+1 \quad$ Parallel Perpendicular Neither
e. $y=4 x+1 \quad$ and $y=\frac{-1}{4} x \quad$ Parallel $\quad$ Perpendicular Neither
f. $3 x+2 y=6$ and $\quad y=\frac{2}{3} x+4 \quad$ Parallel Perpendicular Neither
g. $x=5$ and $\quad y=5 \quad$ Parallel Perpendicular Neither

In each problem, you are given the equation of a line. Create the equation for any line parallel to the first line, and the equation for any line perpendicular to the first line. Finally, graph all three lines on the same coordinate system.

5. Original line: $y=1 / 4 x-2$
a. Parallel line: $\qquad$
b. Perpendicular line: $\qquad$
c. Graph all three lines to the left.

6. Original line: $y=1 / 4 x-2$
a. Parallel line: $\qquad$
b. Perpendicular line: $\qquad$
c. Graph all three lines to the left.

Adult Learning Academy<br>Elementary Algebra Workbook<br>4.3 Systems of Equations

A "system" of equations is just more than one equation that must be true at the same time.

For example, here is a system of two equations: $\quad \mathbf{x}+\mathbf{y}=7$

$$
2 x+y=10
$$

There are infinitely many solutions to the equation $x+y=7$. For example, the points $(0,7),(1,6),(2,5),(3,4),(4,3),(5,2),(6,1),(7,0),(8,-1)$, and $(9,-2)$ all satisfy the equation and are on the line $\mathrm{x}+\mathrm{y}=7$.

Similarly, there are infinitely many solutions to the equation $2 \mathrm{x}+\mathrm{y}=10$. For example, the points ( 0 , $10),(1,8),(2,6),(3,4),(4,2),(5,0),(6,-2),(7,-4)$, and $(8,-6)$ all satisfy the equation and are on the line $2 \mathrm{x}+\mathrm{y}=10$.

To solve the SYSTEM of equations means to find the point that satisfies BOTH equations in the system. In other words, we are looking for a point that is on BOTH lines. It's where the lines INTERSECT (cross)!

1. Graph the lines $\mathbf{x}+\mathbf{y}=\mathbf{7}$ and $\mathbf{2 x + y}=\mathbf{1 0}$ below and see if you can find the point where they cross. That's the solution to the system. In future courses you will even learn how to find this point without graphing!


### 4.3 Systems of Equations (cont.)

Use a ruler to carefully graph each pair of lines on the one coordinate system and find their intersection. The point of intersection is the solution to the system of equations.
2.

3.

4.


$$
y=1 / 2 x-1
$$

$$
y=-x-7
$$

Check by plugging the $x$ and $y$ coordinates of your solution into each equation. They should both be satisfied! Did you find the $x$ and $y$ values that make BOTH equations true?
$y=x-7$
$y=-x+3$

Does your solution check?
$y=\frac{2}{3} x-4$
$y=\frac{2}{3} x+1$

What do you notice about these lines? Where do they cross?

## Adult Learning Academy Elementary Algebra Workbook 4.4 GRAPHING LINEAR INEQUALITIES

a. $\mathrm{X}+\mathrm{Y}=7$



d. $\mathrm{X}+\mathrm{Y} \leq 7$
e. $\quad \mathrm{X}+\mathrm{Y} \geq 7$
f. $\quad Y>2 x-5$




## RECIPE for graphing linear inequalities:

1. Temporarily pretend that there is an equal sign.
2. Graph the border line: use a solid line for $\geq$ or $\leq$, use a dotted line for $>$ or $<$.
3. Choose a test point NOT on the border. Most people choose $(0,0)$ as long as it's not on the border.
4. Plug the x-coordinate and the y-coordinate of the test point into the original inequality. If the point satisfies the inequality, then shade the side containing that point. If the point does NOT satisfy the inequality, then shade the other side.

### 4.4 GRAPHING LINEAR INEQUALITIES (CONT.)

g. $y<1 / 4 x-2$

j. $\quad y \geq 7$

m. $y>-3 / 4 x+2$

h. $y>3 x$

k. $y<-2 / 3 x+1$

n. $y-x \leq 3$

i. $\mathrm{y} \leq 1 / 2 \mathrm{x}+3$

l. $\mathrm{x}<4$

o. $3 x+5 y<15$


## POINT-SLOPE FORM

(to the tune of Ghostbusters), lyrics by Debbie Char

> When you've got a point
> And you know the slope,
> Whatcha gonna use?
> POINT-SLOPE FORM!!!

> When you've got TWO points, You can find the slope, then Whatcha gonna use? POINT-SLOPE FORM!!!

Y minus $\mathrm{Y}_{1}$, which is the y-coordinate, Equals m, which is the slope, then in parentheses X minus $\mathrm{X}_{1}$, which is the x -coordinate, That's the Point-Slope form of the equation of a line!

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Adult Learning Academy
Elementary Algebra Workbook
4.6 Point-Slope Form Practice

$$
\mathbf{y}-\mathbf{y}_{1}=\mathbf{m}\left(\mathbf{x}-\mathbf{x}_{1}\right)
$$

|  | WHAT YOU KNOW |
| :--- | :--- | :--- | :--- | :--- | :--- |

4.6 Point-Slope Form Practice (CONT.)


Adult Learning Academy<br>Elementary Algebra Workbook<br>4.7 CAREER APPLICATIONS: STEM

1. A farmer starts with 40 acres, and plants two additional acres each year.
a. Graph the number of acres planted over the years. Why is it only shown in the first quadrant?

b. What is the slope of this line? Why is it positive? What is the $y$-intercept?
c. Write the equation of the line. What form of the linear equation did you choose?
d. Another farmer starts at 44 acres, and also plants 2 additional acres each year. Make this line in another color on the same graph.
e. Write the equation for the second farmer's acres planted. What do you notice about the two lines? How can you tell that this is true from their equations?
f. A third farmer starts with 40 acres planted, but each year plants 3 more. Graph her line in a third color.
g. Compare this new line with the other two. What is the same? What is different? Explain what you see.
h. If all three farmers continue in their patterns for 30 years, how much will each be planting in their $30^{\text {th }}$ year?
2. A generous long-term grant gives a research team $\$ 100$ million. Each year they spend 5 million dollars of the grant.
a. Graph the amount left on the grant over the first six years:

b. What is the slope of this line? Is it positive or negative? Why?
c. Write the equation for the line. What form did you choose?
d. Another lab received a grant for $\$ 80$ million, and will also use $\$ 5$ million per year. Write the equation for the amount of money left in the grant each year, and graph it in another color above. What can you say about the two lines?
e. A third lab received a grant for $\$ 100$ million, but will use $\$ 10$ per year. Write the equation for its value over time, and graph it in another color above. Compare this equation to the others. What is the same? What is different?
f. After 10 years, how much of each grant will remain?
3. A researcher who has been with the lab 5 years makes $\$ 55,000$ per year. A researcher who has been with the lab 7 years makes $\$ 65,000$ per year.
a. Use the two points described above to graph a line:

b. What is the slope of this line?
c. Write the equation of the line. What form did you use?
d. According to your equation, how much would a researcher make annually after working 25 years?
e. According to your equation, how much would a starting researcher make?
4. A lab can hire a maximum of 20 technicians. Some can be part-time and others can be full-time. If $x=$ the number of part-timers and $y=$ the number of full-timers, the inequality $x+y \leq 20$ describes the situation.
a. Graph all possible combinations of part-time and full-time drivers:
b. Can the lab hire 10 part-time and 10 full-time technicians? Where is this point?

c. Can the company hire 5 part-time and 5 full-time technicians? Where is this point?
d. Can the company hire 18 part-time and 5 full-time technicians? Where is this point?
5. The Human Resources Department at work offers you a choice of two prescription drug plans. Plan A charges $\$ 50$ per year plus $\$ 10$ per prescription filled. Prescription Plan B charges $\$ 25$ per year plus $\$ 15$ per prescription filled.
a. Graph and label each equation.

$$
\begin{aligned}
& \mathrm{A}(\mathrm{x})=50+10 \mathrm{x} \\
& \mathrm{~B}(\mathrm{x})=25+15 \mathrm{x}
\end{aligned}
$$


b. Where do the two lines cross? Interpret the meaning of that point.

Adult Learning Academy

Elementary Algebra Workbook
Module 4 Answer Key

### 4.1 How to Write the Equation of a Line

1a. $y=-1 / 2 x+3$
1b. $y=3 / 4 x-2$

2a. $y-7=-1 / 2(x-4)$
2b. $y-5=3 / 4(x+2)$

3a. $m=\frac{1-7}{-2-3}=\frac{-6}{-5}=6 / 5$

$$
y-7=6 / 5(x-3) \quad \text { OR } \quad y-1=6 / 5(x+2)
$$

3a. $\mathrm{m}=\frac{-3-5}{2-0}=\frac{-8}{2}=-4$
$y-5=-4 x \quad$ OR $\quad y+3=-4(x-2)$

### 4.2 Parallel and Perpendicular Lines

1. 


4a. Parallel
4b. Neither
4c. Perpendicular
4d. Neither
4e. Perpendicular
4f. Perpendicular
4g. Perpendicular

2a. Yes
2b. No
2c. No
2d. Yes
2e. Yes
2f. No
3.


5a. may vary: $y=1 / 4 x+3$ (slope must equal $1 / 4$ )
5b. may vary: $\mathrm{y}=-4 \mathrm{x}+1$ (slope must equal -4)

5c.

3. $(5,-2)$

4. Parallel lines; do not cross


### 4.4 Graphing Linear Inequalities

a.

e.

i.

m.

n.


### 4.6 Point-Slope Practice

a. $y-1=1 / 2(x-3)$

e. $y-1=3 / 4(x-0) O R$ $y=3 / 4 x+1$

b. $y-4=-2 / 3(x+5)$

f. $\quad m=3 / 4$
$y-5=3 / 4(x-3)$ OR $y-2=3 / 4(x+1)$

0.

c.

g.

k.

c. $y+4=2(x+3)$

g. $m=-1$
$y-3=-1(x+4) O R$ $y+2=-1(x-1)$ OR $y=-x-1$

d. $y-1=0(x-3) O R$ $y=1$

h. $y+3=2 x-2$
$y=2 x-5$


### 4.6 Point-Slope Practice (cont.)

i. $y+3=2 x-2$
$y=2 x-5$

j. $y-2=1 / 2 x+3$
$y=1 / 2 x+5$


### 4.7 Career Applications: STEM

1a. Acres and time only make sense in positive quantities


1b. $m=2, y$-int. $=40$; slope is positive because the more years go by, the more acres are planted
1c. slope-intercept form $y=2 x+40$
1d. see graph above
1e. $y=2 x+44$; lines a and $d$ are parallel because they have the same slope
1f. $y=3 x+40$
1g. has same $y$-intercept as line $a$, but has steeper incline
1h. $y=2(30)+40=100$ acres
$y=2(30)+44=104$ acres
$y=3(30)+40=130$ acres

2a.


2b. $m=-5 / 1$; as years increase, money decreases
2c. slope-intercept form $y=-5 x+100$
k. $y+1=3 x+6$
$y=3 x+5$


2d. $y=-5 x+80$; lines a and $c$ are parallel because they have the same slope
2e. $y=-10 x+100$; same $y$-intercept as line a but steeper decline
2f. $y=-5(10)+100=\$ 50$ million left
$y=-5(10)+80=\$ 30$ million left
$y=-10(10)+100=\$ 0$ left
3a.


3b. $m=\frac{65,000-55,000}{7-5}=\frac{10,000}{2}=5,000$
3c. point slope form $y-65,000=5,000(x-7) O R$ point slope form $y-55,000=5,000(x-5) O R$ slope-intercept form $\mathrm{y}=5000 \mathrm{x}+\mathbf{3 0 , 0 0 0}$
3d. $\mathrm{y}=5000(25)+\mathbf{3 0 , 0 0 0}=\$ 155,000$
3e. $y=5000(0)+30,000=\$ 30,000$
4a. $\mathrm{y} \leq-\mathrm{x}+20$


4b. Yes, the point is on the line
4c. Yes, this point is in the shaded area
4d. No, this point is not in the shaded area

### 4.7 Career Applications: STEM (cont.)

5 a.


5 . They intersect at $(5,100)$; if you get 5 prescriptions, the cost will be the same with either plan

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Adult Learning Academy Elementary Algebra Workbook

## Module 5: Integer Exponents \& Laws, Scientific Notation, Polynomials \& Operations

## Learning Objectives

By the time you finish this module, you should be able to:
$\square$ Simplify expressions involving positive or negative exponents according to the exponent rulesConvert a number in standard notation into scientific notationConvert a number in scientific notation into standard notationIdentify a monomial, binomial, trinomial, and polynomialSort polynomials according to their degreeAdd, subtract, and multiply polynomials of any sizeDivide a polynomial by a monomial

## IMPORTANT INFORMATION FROM MODULE 5:

$x^{m} \cdot x^{n}=x^{m+n}$
$\left(x^{m}\right)^{n}=x^{m n} \quad$ "when you raise a power to a power, mutiply the powers"
$\frac{x^{m}}{x^{n}}=x^{m-n}$
$x^{0}=1,0^{m}=0,0^{0}$ is undefined
$\left(\frac{x}{y}\right)^{m}=\frac{x^{m}}{y^{m}} \quad x^{-m}=\frac{1}{x^{m}} \quad$ A negative exponent does NOT make a number negative!
one term: monomial; two terms: binomial; three terms: trinomial
To find the degree of a single term, add all the exponents on all the variables in that term. To find the degree of a polynomial, have a contest: the term with the highest degree wins!

FOIL: for multiplying a binomial times a binomial (also for squaring a binomial!!)
$(a+b)^{2}=a^{2}+2 a b+b^{2} \quad(a-b)^{2}=a^{2}-2 a b+b^{2} \quad(a+b)(a-b)=a^{2}-b^{2}$
Birthday Song: You must have like terms, you must have like terms to ADD or SUBTRACT, you must have like terms!

When you subtract a polynomial, be sure to subtract EVERY term!!

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Adult Learning Academy
Elementary Algebra Workbook
Module 5 Video \& ExERCISE LIST

| Topic | Website | Videos | Exercises |
| :---: | :---: | :---: | :---: |
| Exponents | www.khanacademy.org | Level 1 Exponents | Positive and Zero Exp |
|  |  | Understanding Exponents 2 | Negative Exponents |
|  |  | Understanding Exponents | Exponent Rules |
|  |  | Level 2 Exponents (negative exp) |  |
|  |  | Exponent Rules Part 1 |  |
|  |  | Exponent Rules Part 2 |  |
|  |  | Exponent Prop involving Quotients |  |
|  | www.stlcc.edu | Exponent Rules ppt on Blackboard |  |
|  | http://www.youtube.com/watch?v=h063AzwjGlc | Mathman: 3 Exponent Mistakes |  |
| Scientific Notation | www.khanacademy.org | Scientific Notation | Scientific Notation |
|  |  | Scientific Notation 1 |  |
| Polynomials | http://www.youtube.com/watch?v=D-3NIysYshM | Diff betw Trinom, Bi, Monomial |  |
|  | http://www.youtube.com/watch?v=l_kY3sHViSA | Identifying Degree, Name of Polyn. |  |
|  | www.khanacademy.org | Tems Coefficients and Exponents |  |
|  |  | Evaluating a Polynomial at a Given V |  |
|  |  | Simplify a Polynomial |  |
| Add, Subt. Polynom. | www.khanacademy.org | Adding Polynomials | Adding, Sub Polynom. |
|  |  | Ex: Adding Polynomials w/Mult Var. |  |
|  |  | Add \& Subt of Polynomials |  |
|  |  | Adding and Sub Polynomials 1 |  |
|  |  | Adding \& Subt Polynomials 2 |  |
|  |  | Adding and Sub Polynomials 3 |  |
|  |  | Subtracting Polynomials |  |
|  |  | Sub Polynomials w/ Mult Variables |  |
|  |  |  |  |


| Topic | Website | Videos | Exercises |
| :--- | :--- | :--- | :--- |
| Multiplying Polynom. | www.khanacademy.org | Multiplying Monomials | Multip. Express. 0.5 |
|  |  | Multiplying Monomials by Polynom. | Multiplying Exp. 1 |
|  |  | Multiplying Binomials | Multiplying Polynom. |
|  |  | Multiplying Polynomials1 |  |
|  |  | Multiplication of Polynomials |  |
|  |  | Square a Binomial |  |
|  |  | Special Products of Binomials |  |
|  |  | Special Polynomials Products 1 |  |
|  |  | Special Products of Polynomials 1 |  |
| Dividing Polynomials | www.khanacademy.org | Multiplying Polynomials |  |
|  |  | More Multiplying Polynomials |  |
| Module 5 Test Review | www.stlcc.edu | Polynomial Divided by Monomial |  |
|  |  | Dividing Multivariable Poly. w/ Mono |  |
|  |  |  | Exponent Rules |
|  |  |  | Module 5 Review Flashcards |

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Adult Learning Academy
Elementary Algebra Workbook
5.1 Simplifying Expressions

1. $x^{6} \cdot x^{2}$
2. $x^{6}+x^{2}$
3. $x^{6} \div x^{2}$
4. $\left(x^{6}\right)^{2}$
5. $10 x^{8} \cdot 2 x^{2}$
6. $\left(10 x^{8}\right)^{2}$
7. $\frac{10 x^{8}}{2 x^{2}}$
8. $10 x^{8}-2 x^{2}$
9. $10 x^{8}-2 x^{8}$
10. $-5^{2}$
11. $5^{-2}$
12. $0^{5}$
13. $0^{0}$
14. $-5^{-2}$
15. $(-5)^{2}$
16. $(-5)^{-2}$
17. $5 x^{-2}$
18. $(5 x)^{-2}$
19. $x^{0}$
20. $5 x^{0}$
21. $(5 x)^{0}$
22. $\left(\frac{5}{x}\right)^{-2}$
23. $\frac{12 x^{4} x^{8}}{4 x^{3}}$
24. $\frac{-10 x^{5} y^{-3}}{15 x^{-3} y^{2}}$
25. $\frac{4 x^{7} x^{-3} y^{7}}{4 x^{5} y^{6}}$
26. $\left(\frac{7 x^{5} y^{-2}}{14 x^{-3} y^{4}}\right)^{3}$
27. $\left(\frac{12 x^{-2} y^{4}}{4 x^{-3} y^{-3}}\right)^{-3}$

Adult Learning Academy
Elementary Algebra Workbook
5.2 Color Matching Simplified Expressions

## Simplify each expression, and color the matching simplified expressions.



|  | Adult Learning Elementary Algebra 5.3 Multiplying Po |  |  |
| :---: | :---: | :---: | :---: |
| 1. $x \cdot x$ | 11. $(\mathrm{x}+7)(\mathrm{x}-5)$ | 21. $(3 x-5)^{2}$ |  |
| 2. $\mathrm{x} \cdot \mathrm{x}^{2}$ | 12. $(x+7)(x-5)$ | 22. $(2 x+7)^{2}$ |  |
| 3. $x^{2} \cdot x^{3}$ | 13. $(x+7)(x-7)$ | 23. $(3 x-5)(3 x+5)$ |  |
| 4. $x^{5} \cdot x^{2}$ | 14. $(x-5)(x+5)$ | 24. $(2 x+7)(2 x-7)$ |  |
| 5. $5 x^{3} \cdot-2 x^{4}$ | 15. $(x+7)^{2}$ | 25. $(x+y)(x-y)$ |  |
| 6. $-6 x^{5} \cdot-4 x^{3}$ | 16. $(\mathrm{x}-7)^{2}$ | 26. $(x+y)^{2}$ |  |
| 7. $3 x\left(4 x^{2}-5 x+1\right)$ | 17. $(x+5)^{2}$ | 27. $(\mathrm{x}-\mathrm{y})^{2}$ |  |
| 8. $-2 x^{3}\left(5 x^{4}-3\right)$ | 18. $(\mathrm{x}-5)^{2}$ | 28. $(3 x+2 y)(3 x-2 y)$ |  |
| 9. $7 x^{2}\left(x^{3}-3 x+2\right)$ | 19. $(3 x+5)(2 x-7)$ | 29. $(3 x+2 y)^{2}$ |  |
| 10. $(x+7)(x+5)$ | 20. $(3 x-5)(2 x+7)$ | 30. $(3 x-2 y)^{2}$ |  |

Adult Learning Academy Elementary Algebra Workbook
5.4 ScIENTIFIC NOTATION

Fill in the table:

| Item | Scientific Notation | Standard Notation |
| :---: | :---: | :---: |
| Approximate number of hairs on <br> your head |  | 140,000 |
| Number of cells in your brain |  | $100,000,000,000$ |
| Length of a rhinovirus in meters | $2.7 \times 10^{9}$ | .000000020 |
| Number of heartbeats in a lifetime | $1.0 \times 10^{-8}$ |  |
| Speed that human hair grows in <br> miles per hour | $3.0 \times 10^{13}$ |  |
| Number of red blood cells in a <br> human body | $2.75 \times 10^{-3}$ | $5,000,000$ |
| Lung capacity of a blue whale in <br> milliliters |  |  |
| Speed of a snail in kilometers per <br> second | Thickness of a sheet of paper in <br> inches |  |
| The |  |  |

Adult Learning Academy Elementary Algebra Workbook 5.5 THINKING AbOUT POLYNOMIALS

| Expression | CHOOSE: Monomial, Binomial, Trinomial, Polynomial | Degree |
| :---: | :---: | :---: |
| $3 x^{2}-2 x+1$ |  |  |
| $5 x y z$ |  |  |
| $4 x+2 y$ |  |  |
| $5 x^{2} y-11$ |  |  |
| 139 |  |  |
| $-642 x^{39}$ |  |  |
| $2 y+3 x-5 w+p$ |  |  |
| $57 \mathrm{x}^{5}-2 \mathrm{x}^{3}+11 \mathrm{x}$ |  |  |
| X |  |  |
| $4 x^{2}+3 x+x$ |  |  |
| $2 \mathrm{x}+\mathrm{y}+\mathrm{z}^{12}$ |  |  |
| $5 x^{0}$ |  |  |
| $100 \mathrm{x}^{2}-\mathrm{py}^{3}$ |  |  |
| $3 x+2$ |  |  |
| $10 x-y+z+p-5$ |  |  |

1. Create a $2^{\text {nd }}$-degree trinomial:
2. Create a $4^{\text {th }}$-degree monomial:

3. Create a $3^{\text {rd }}$-degree binomial:
4. Can the sum of two binomials ever be a trinomial? If so, show an example:
5. Can the sum of two binomials ever be a monomial? If so, show an example:
6. Can the sum of two binomials ever be a binomial? If so, show an example:
7. Can the product of two binomials ever be a binomial? If so, show an example:
8. Can the product of two binomials ever be a trinomial? If so, show an example:
9. Can the product of two binomials ever have four terms? If so, show an example:

Adult Learning Academy
Elementary Algebra Workbook

### 5.6 Evaluating, Adding, and Subtracting <br> Polynomials

Here are three functions:

$$
f(x)=3 x^{2}-2 x+1 \quad g(x)=-2 x^{2}-5 \quad h(x)=-4 x+2
$$

To "evaluate" means to plug in the value of x and see what you get. When you plug in a negative number for x , always put parentheses around it!

1. Evaluate the following:
a) $f(2)$
b) $g(-3)$
c) $\mathrm{h}(0)$
d) $\mathrm{f}(-5)$
e) $g(0)$
f) $h(-5)$
2. When you add and subtract polynomials, combine like terms. When you subtract, be sure to subtract EVERY term!
a) $f(x)+g(x)$
b) $g(x)+h(x)$
c) $f(x)+h(x)$
d) $f(x)-g(x)$
d) $f(x)-h(x)$
e) $g(x)-h(x)$
3. You can also multiply every term of a polynomial by a number. Find these:
a) $4 f(x)+3 g(x)$
b) $5 f(x)-2 h(x)$
4. The day's revenue (income) for a computer company depends on how many clients come to get their computers fixed. The revenue can be modeled by the function

$$
\begin{array}{ll}
R(x)=x^{3}-x^{2} \quad \begin{array}{l}
\text { where } \mathrm{R}(\mathrm{x}) \text { is the revenue in dollars, } \\
\text { and } \mathrm{x} \text { is the number of clients that day }
\end{array}
\end{array}
$$

The day's costs (expenses) for the same company also depend on how many clients come to get their computers fixed. The costs can be modeled by the function

$$
\begin{aligned}
C(x)=.75 x^{3}-.7 x^{2}-.5 x+10 & \text { where } \mathrm{C}(\mathrm{x}) \text { is the cost in dollars, } \\
& \text { and } \mathrm{x} \text { is the number of clients that day }
\end{aligned}
$$

Any business calculates its PROFIT by starting with Revenue (income) and subtracting expenses (costs). So the profit function, $\mathrm{P}(\mathrm{x})$, can be modeled by

$$
P(x)=R(x)-C(x) .
$$

a. For this company, what is the profit function? (subtract $R(x)-C(x))$ :
b. If 3 clients come to get their computers fixed on a given day, what is the Revenue? The Cost? The Profit?
c. If 20 clients come to get their computers fixed on a given day, what is the Revenue? The Cost? The Profit?
d. Say that Revenue doubles (get multiplied by 2), but costs remain the same. Show the new function for Profit:
2. Write an algebraic expression for the PERIMETER (add all the sides) and the AREA (length times width) of each rectangle on the page. Remember that area is measured in square units.
DIMENSIONS

a. | PERIMETER |
| :--- | AREA

x inches
x inches
b.

$2 x$ feet +1
c.


3 xcm
d.

e.

f.

3. Barnes-Jewish Hospital in St. Louis is 177 feet tall. If you stood at the top of the hospital and dropped a penny (NOTE: this is NOT recommended!), the following function tells you high off the ground the penny would be after t seconds:

$$
h(t)=-16 t^{2}+177
$$

where $t$ is the number of seconds since you
 dropped the penny, and $h(t)$ is the penny's height in feet off the ground.
a. Find $\mathrm{h}(0)$. What does this information tell you?
b. Find $h(1)$, the height of the penny 1 second after being dropped:
c. Find $\mathrm{h}(2)$, the height of the penny 2 seconds after being dropped:
d. Find $h(3)$, the height of the penny 3 seconds after being dropped:
e. Would the penny still be falling 4 seconds after being dropped? How do you know?
5.2 Color Matching Expressions

$$
\begin{aligned}
& 2^{0}=\frac{3 x}{3 x}=1 \\
& \frac{2 x}{x}=\left(53 x^{4}+3\right)-\left(53 x^{4}+1\right)=2 x^{0} \\
& -5^{2}=0 x-25=-25 \\
& \hline x+x=5 x-3 x=2 x \\
& \hline 100 x^{2}-99 x^{2}=x \cdot x=x^{2} \\
& \hline 3 x^{2}-x^{2}=\frac{10 x^{5}}{5 x^{3}}=2 x^{2} \\
& \hline \frac{x^{3}+x}{x}=x^{2}+1=\left(5 x^{2}-1\right)-\left(4 x^{2}-2\right)=x^{2}+1 \\
& \hline(x+1)(x-1)=\left(5 x^{2}-2\right)-\left(4 x^{2}-1\right)=x^{2}-1 \\
& \hline(x+1)^{2}=x^{2}+2 x+1 \\
& \hline(x-1)^{2}=\frac{5 x^{2}-10 x+5}{5}=x^{2}-2 x+1 \\
& \hline
\end{aligned}
$$

### 5.3 Multiplying Polynomials

1. $x^{2}$
2. $x^{3}$
3. $x^{5}$
4. $x^{7}$
5. $-10 x^{7}$
6. $24 x^{8}$
7. $12 x^{3}-15 x^{2}+3 x$
8. $-10 x^{7}+6 x^{3}$
9. $7 x^{5}-21 x^{3}+14 x^{2}$
10. $x^{2}+12 x+35$
11. $x^{2}+2 x-35$
12. $x^{2}-12 x+35$
13. $x^{2}-49$
14. $x^{2}-25$
15. $x^{2}+14 x+49$
16. $x^{2}-14 x+49$
17. $x^{2}+10 x+25$
18. $x^{2}-10 x+25$
19. $6 x^{2}-11 x-35$
20. $6 x^{2}+11 x-35$
21. $9 x^{2}-30 x+25$

### 5.3 Multiplying Polynomials (cont.)

22. $4 x^{2}+28 x+49$
23. $9 x^{2}-25$
24. $4 x^{2}-49$
25. $x^{2}-y^{2}$
26. $x^{2}+2 x y+y^{2}$
27. $x^{2}-2 x y+y^{2}$
28. $9 x^{2}-4 y^{2}$
29. $9 x^{2}+12 x y+4 y^{2}$
30. $9 x^{2}-12 x y+4 y^{2}$

### 5.4 Scientific Notation

| Item | Scientific <br> Notation | Standard Notation |
| :---: | :---: | :---: |
| Number of hairs on <br> your head | $\mathbf{1 . 4 \times \mathbf { 1 0 } ^ { 5 }}$ | 140,000 |
| Number of cells in <br> your brain | $\mathbf{1 . 0 \times 1 0 ^ { 1 1 }}$ | $100,000,000,000$ |
| Length of a <br> rhinovirus in meters | $\mathbf{2 . 0 \times 1 0 ^ { - 8 }}$ | .000000020 |
| Number of <br> heartbeats in a <br> lifetime | $2.7 \times 10^{9}$ | $\mathbf{2 , 7 0 0 , 0 0 0 , 0 0 0}$ |
| Speed that human <br> hair grows in miles <br> per hour | $1.0 \times 10^{-8}$ | $\mathbf{. 0 0 0 0 0 0 0 1}$ |
| Number of red <br> blood cells in a <br> human body | $3.0 \times 10^{13}$ | $\mathbf{3 0 , 0 0 0 , 0 0 0 , 0 0 0 , 0 0 0}$ |
| Lung capacity of a <br> blue whale in mL | $\mathbf{5 . 0 \times 1 0 ^ { 6 }}$ | $5,000,000$ |
| Speed of a snail in <br> kilometers per <br> second | $\mathbf{1 . 3 \times 1 0 ^ { - 5 }}$ | .000013 |
| Thickness of a sheet <br> of paper in inches | $2.75 \times 10^{-3}$ | $\mathbf{. 0 0 2 7 5}$ |

### 5.5 Thinking About Polynomials

| Expression | CHOOSE: | Degree |
| :---: | :---: | :---: |
| $3 \mathrm{x}^{2}-2 \mathrm{x}+1$ | Trinomial | $\mathbf{2}^{\text {nd }}$ |
| 5 xyz | Monomial | $\mathbf{3}^{\text {rd }}$ |
| $4 \mathrm{x}+2 \mathrm{y}$ | Binomial | $\mathbf{1}^{\text {st }}$ |
| $5 \mathrm{x}^{2} \mathrm{y}-11$ | Binomial | $\mathbf{3}^{\text {rd }}$ |
| 139 | Monomial | $\mathbf{0}^{\text {degree }}$ |
| $-642 \mathrm{x}^{39}$ | Monomial | $\mathbf{3 9}^{\text {th }}$ |
| $2 \mathrm{y}+3 \mathrm{x}-5 \mathrm{w}+\mathrm{p}$ | Polynomial | $\mathbf{1}^{\text {st }}$ |
| $57 \mathrm{x}^{5}-2 \mathrm{x}^{3}+11 \mathrm{x}$ | Trinomial | $\mathbf{5}^{\text {th }}$ |
| x | Monomial | $\mathbf{1}^{\text {st }}$ |
| $4 \mathrm{x}^{2}+3 \mathrm{x}+\mathrm{x}$ | Trinomial | $\mathbf{2}^{\text {nd }}$ |
| $2 \mathrm{x}+\mathrm{y}+\mathrm{z}^{12}$ | Trinomial | $\mathbf{1 2}^{\text {th }}$ |
| $5 \mathrm{x}^{0}$ | Monomial | $\mathbf{0}^{\text {degree }}$ |
| $100 \mathrm{x}^{2}-\mathrm{py}^{3}$ | Binomial | $\mathbf{4}^{\text {th }}$ |
| $3 \mathrm{x}+2$ | Binomial | $\mathbf{1}^{\text {st }}$ |
| $10 \mathrm{x}-\mathrm{y}+\mathrm{z}+\mathrm{p}-5$ | Polynomial | $\mathbf{1}^{\text {st }}$ |

### 5.5 Thinking About Polynomials (cont.)

1. answers will vary, ex. $x^{2}+3 x-5$
2. answers will vary, ex. $5 \mathrm{x}^{4}$
3. answers will vary, ex. $5 x^{3}-3 x$
4. Yes, $(x+3)+(y+5)=x+y+8$
5. Yes, $(x+3)+(x-3)=2 x$
6. Yes, $(x+3)+(x+5)=2 x+8$
7. Yes, $(x+3)(x-3)=x^{2}-9$
8. Yes, $(x+3)(x+2)=x^{2}+5 x+6$
9. Yes, $(x+3)(y+w)=x y+x w+3 y+3 w$

### 5.6 Evaluating, Adding, and Subtracting Poly.

1a. $f(2)=3(4)-2(2)+1=9$
1b. $g(-3)=-2(9)-5=-23$
1c. $h(0)=-4(0)+2=2$
1d. $\mathrm{f}(-5)=3(25)-2(-5)+1=86$
1e. $g(0)=-2(0)-5=-5$
1f. $h(-5)=-4(-5)+2=22$
2a. $x^{2}-2 x-4$
2b. $-2 x^{2}-4 x-3$
2c. $3 x^{2}-6 x+3$
2d. $5 x^{2}-2 x+6$
2e. $-2 x^{2}+4 x-7$
2f. $3 x^{2}+2 x-1$
3a. $12 x^{2}-8 x+4-6 x^{2}-15$ $=6 x^{2}-8 x-11$
3b. $15 x^{2}-10 x+5+8 x-4$
$=15 x^{2}-2 x+1$
5.7 Career Applications: STEM

1a. $R(x)-C(x)=x^{3}-x^{2}-\left(.75 x^{3}-.7 x^{2}-.5 x+10\right)$

$$
\begin{aligned}
& =x^{3}-x^{2}-.75 x^{3}+.7 x^{2}+.5 x-10 \\
& =.25 x^{3}-.3 x^{2}+.5 x-10
\end{aligned}
$$

1b. $\mathbf{R ( 3 )}=3^{3}-3^{2}=27-9=\$ 18$
$\mathbf{C}(3)=.75\left(3^{3}\right)-.7\left(3^{2}\right)-.5(3)+10$
$=.75(27)-.7(9)-1.5+10$
= 20.25-6.3-1.5 + 10
$=\$ 22.45$
$\mathbf{P}(3)=18-22.45=-\$ 4.45$ (loss)
1c. $\mathbf{R}(\mathbf{2 0})=20^{3}-20^{2}=8000-400=\$ 7600$
$\mathbf{C}(\mathbf{2 0})=.75\left(20^{3}\right)-.7\left(20^{2}\right)-.5(20)+10$
$=.75(8000)-.7(400)-10+10$
$=6000-280$
= \$5720
$\mathbf{P ( 2 0 )}=7600-5720=\$ \mathbf{1 8 8 0}$

### 5.7 Career Applications: STEM (cont.)

1d. $\mathbf{P}(x)=2 R(x)-C(x)$
$=2 \mathrm{x}^{3}-2 \mathrm{x}^{2}-\left(.75 \mathrm{x}^{3}-.7 \mathrm{x}^{2}-.5 \mathrm{x}+10\right)$
$=1.25 x^{3}-1.3 x^{2}+.5 x-10$

2a. $P=x+x+x+x=4 x$ in.
A $=x * x=x^{2}$ sq. in.
2b. $\mathbf{P}=\mathrm{x}+(2 \mathrm{x}+1)+\mathrm{x}+(2 \mathrm{x}+1)$

$$
=6 \mathrm{x}+2 \mathrm{ft} .
$$

$$
\mathbf{A}=x(2 x+1)
$$

$$
=2 x 2+x \text { sq. ft. }
$$

2c. $\mathbf{P}=3 \mathrm{x}+(\mathrm{x}-5)+3 \mathrm{x}+\mathrm{x}-5$

$$
=8 \mathrm{x}-10 \mathrm{~cm}
$$

$$
A=3 x(x-5)
$$

$$
=3 x 2-15 x \text { sq. cm }
$$

2d. $\mathbf{P}=(x+5)+(x-3)+(x+5)+(x-3)$

$$
=4 x+4 \text { meters }
$$

A $=(\mathrm{x}+5)(\mathrm{x}-3)$
$=x^{2}-3 x+5 x-15$

$$
=x^{2}+2 x-15 \text { sq. meters }
$$

2e. $\mathbf{P}=(x+7)+(x-7)+(x+7)+(x-7)$

$$
=4 x \text { miles }
$$

A $=(x+7)(x-7)$
$=x^{2}-49$ sq. miles
2f. $\mathbf{P}=(x+5)+(x+5)+(x+5)+(x+5)$

$$
=4 x+20 \mathrm{~km}
$$

$$
\begin{aligned}
A & =(x+5)(x+5) \\
& =x^{2}+\mathbf{1 0 x}+\mathbf{2 5} \text { sq. } \mathbf{k m}
\end{aligned}
$$

3a. $\mathbf{h}(\mathbf{0})=-\mathbf{1 6}(\mathbf{0})^{2}+177=177$
This tells us that the penny is 177 feet off the ground (on top of the building) when you haven't thrown it yet
3b. $\mathbf{h ( 1 )}=-16(1)^{2}+177=-16+177=\mathbf{1 6 1}$ feet.
3c. $\mathbf{h ( 2 )}=-16(2)^{2}+177=-64+177=\mathbf{1 1 3}$ feet
3d. $\mathbf{h ( 3 )}=-16(3)^{2}+177=\mathbf{3 3}$ feet
3e. No ; $\mathrm{h}(4)=-16(4)^{2}+177=-79$ feet (underground?)!

Adult Learning Academy Elementary Algebra Workbook

Module 6: Factoring Polynomials, solving quadratic equations by factoring

## Learning Objectives

By the time you finish this module, you should be able to:

Factor polynomials, choosing from a variety of strategies:
o Greatest Common Factor
o Reverse Foil
o Difference of Two Squares
o Factoring by Grouping
o Difference of Cubes
o Sum of Cubes

Recognize Prime polynomials
$\square$ Solve quadratic equations by factoring
$\square$ Create quadratic equations to model situations and solve application problems

## IMPORTANT INFORMATION FROM MODULE 6:

$a^{2}-b^{2}=(a-b)(a+b)$
$a^{2}+b^{2}$ is prime
$a^{3}+b^{3}=(a+b)\left(a^{2}-\mathrm{ab}+b^{2}\right)$
$a^{3}-b^{3}=(a-b)\left(a^{2}+a b+b^{2}\right)$
ALWAYS check for COMMON FACTORS!
If a polynomial doesn't factor, then it's prime.
Always check inside parentheses to be sure you have factored completely.
Check your factoring by multiplying your factors back to get the original polynomial.
A quadratic equation contains an $x 2$. It has up to two solutions. The solutions are also the $x$-intercepts of the parabola if you graph the equation. To solve a quadratic equation by factoring:
-- Get 0 on one side of the equation
-- Factor the other side
-- Set each factor $=$ to 0
-- Solve

| St. Louis Community College | Adult Learning Academy <br> Elementary Algebra Workbook Module 6 Video \& ExERCISE LIST |  |  |
| :---: | :---: | :---: | :---: |
| Topic | Website | Videos | Exercises |
| Factoring with the | www.khanacademy.org | Factoring/distributive property 1 | (refer to workbook) |
| distributive property |  | Factoring/distributive property 2 |  |
|  |  | Factoring/distributive property 3 |  |
|  |  | Factoring by grouping |  |
| Factoring Quadratic | www.khanacademy.org | Factoring a Quadratic Expression | Factoring Quadratic Expressions |
| Expressions |  | Factoring a Quadratic Expression 2 | More factoring trinomials |
|  | youtube.com | Recognizing a Prime trinomial |  |
|  |  |  |  |
| Factoring Special | www.khanacademy.org | Factoring Difference of Squares | Factoring Difference of Squares |
| Products |  | Factoring Difference of Squares 2 | More Factoring Diff. of Squares |
|  |  | Factoring Perfect Square Trinomials | Factoring Perfect Square Trinomials |
|  | youtube.com | Factoring Rap |  |
|  |  |  |  |
| Factoring trinomials in | youtube.com | Factoring by trial and error | Factoring by grouping |
| the form $\mathrm{ax}^{2}+\mathrm{by}+\mathrm{c}$ | www.khanacademy.org | Factoring by grouping (3 videos) |  |
|  | youtube.com | Factoring Completely |  |
|  |  |  |  |
| Factoring Cubes | Khanacademy.org | Difference of Cubes | Refer to workbook |
|  |  | Sum of Cubes |  |
|  |  | Difference and Sum of Cubes |  |
|  |  |  |  |
| Quadratic Equations | youtube.com | Zero Property Rule | Solving quadratic equations by factoring |
|  | Khanacademy.org | Solving Quadratic Equations by factoring |  |
| Module 6 Test Review | stlcc.edu | Blackboard PowerPoint |  |
|  |  |  |  |

1. Find the GCF from the list of terms.
a. $30,75,135$
d. $12 x^{3}, 6 x^{4}, 3 x^{5}$
b. $x^{3}, x^{2}, x^{5}$
e. $p^{7} q, p^{8} q^{2}, p^{9} q^{3}$
c. $32 x^{5}, 18 x^{2}$ $\qquad$ f. $x^{3}, y^{3}, z^{3}$
2. Factor out the GCF from each polynomial. The first one is done for you.
a. $3 a+6$ $\qquad$ e. $6 x^{3}-9 x^{2}+12 x$
b. $12 \mathrm{x}+4$ $\qquad$ f. $4 x-8 y+4$
c. $42 \mathrm{y}-7$ $\qquad$ g. ${ }^{2} y^{2}+x y^{3}+x^{3} y$
d. $15 \mathrm{a}^{3}+5 \mathrm{a}$ $\qquad$ h. $99 q^{7}+24 q$
i. $a^{7} b^{6}+a^{3} b^{2}-a^{2} b^{5}-a^{2} b^{2}$ $\qquad$
$\qquad$
$\qquad$
3. Since $5 \cdot 4=20$, the numbers 5 and 4 are called $\qquad$ of 20.
4. List all the factors of $9 x y^{2}$ (hint: there are 14 factors)
$\qquad$
5. List all the factors of $7 \mathrm{y}^{3}$

## To factor by grouping:

1. Group the terms in two groups so that each group has a common factor.
2. Factor out the GCF from each group.
3. If there is a common binomial factor, factor it out.
4. If not, rearrange the terms and try again.

$$
\text { Example: Factor } \mathrm{xy}+2 \mathrm{x}+3 \mathrm{y}+6
$$

1. Group terms: $(x y+2 x)+(3 y+6)$
2. Factor out the GCF: $x(y+2)+3(y+2)$
3. Factor out the binomial: $(\mathbf{y}+2)(\mathbf{x}+3)$

## Factor by grouping:

1. $\mathrm{x}^{3}+2 \mathrm{x}^{2}+5 \mathrm{x}+10$
2. $\mathrm{x}^{3}+4 \mathrm{x}^{2}+3 \mathrm{x}+12$
3. $2 y-8+x y-4 y$
4. $5 x+15+x y+3 y$
5. $x y+3 x+3 y+9$
6. $6 x^{3}-4 x^{2}+15 x-10$
7. $a b-5 a+6 b-$

## Elementary Algebra Workbook

6.3 FACTORING TRINOMIALS IN THE FORM OF $\mathbf{X}^{2}+\mathbf{B X}+\mathbf{C}$

1. Factor each trinomial. If the trinomial cannot be factored, write prime. Check your answer by using foil.
a. $\mathrm{x}^{2}+7 \mathrm{x}+12$
b. $y^{2}+11 y+18$
c. $a^{2}+5 a-24$
d. $n^{2}-6 n+5$
e. $\mathrm{w}^{2}-5 \mathrm{w}-50$
f. $a^{2}+3 a+11$
g. $b^{2}+17 b+66$
h. $x^{2}-x-10$
i. $x^{2}+5 x y+6 y^{2}$
j. $\quad x^{4}+5 x^{2}+6$
k. k. $40-13 \mathrm{t}+\mathrm{t}^{2}$
2. $-24+5 a+a^{2}$
3. Write a trinomial that is not prime: $\qquad$
4. Write a trinomial that is prime: $\qquad$
5. Find all positive values of $b$ so that the trinomial is factorable: $y^{2}+b y+20$
6. Factor each trinomial completely. Factor out the GCF first.
a. $2 \mathrm{z}^{2}+20 \mathrm{z}+32$
b. $3 x^{2}+30 x 63$
c. $4 x^{2}-4 x-48$
d. $x^{3}+11 x^{2}+30 x$
7. Write a trinomial that must have its GCF factored out before you can factor.

A perfect square trinomial is a trinomial that is the square of a binomial.
For example: $(\mathrm{x}+5)^{2}=(\mathrm{x}+5)(\mathrm{x}+5)=\mathrm{x}^{2}+10 \mathrm{x}+25$
The following formulas can be used when factoring perfect square trinomials:

$$
\mathbf{a}^{2}+\mathbf{2 a b}+\mathbf{b}^{2}=(\mathbf{a}+\mathbf{b})^{2} \quad \text { and } \quad \mathbf{a}^{2}-\mathbf{2 a b}+\mathbf{b}^{2}=(\mathbf{a}-\mathbf{b})^{2}
$$

1. Factor the following perfect square trinomials. Write your answer with an exponent.
a. $\mathrm{x}^{2}+20 \mathrm{x}+100$
b. $\mathrm{a}^{2}+2 \mathrm{a}+1$
c. $w^{2}-16 w+64$
d. $n^{2}-6 n+9$
e. $x^{2}+2 x y+y^{2}$
f. $9 x^{2}+3 x+1 / 4$

The difference of two squares is another type of special product. Consider the difference between these two perfect squares: $\mathrm{w}^{2}-49=(\mathrm{w}+7)(\mathrm{w}-7)$ Notice when foil is used to multiply the two binomials, the middle term cancels out.

The following formula can be used to factor the difference of squares:

$$
\mathbf{a}^{2}-\mathbf{b}^{2}=(\mathbf{a}+\mathbf{b})(\mathbf{a}-\mathbf{b})
$$

2. Factor the following difference of squares.
a. $\mathrm{w}^{2}-81$
b. $16 a^{2}-1$
c. $\mathrm{b}^{2}-\frac{9}{25}$
d. $c^{4}-d^{6}$
e. $121 \mathrm{x}^{2}-1 / 4$
f. $x^{4} y^{2}-z^{8}$

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Elementary Algebra Workbook
6.5 FACTORING TRINOMIALS IN THE FORM OF AX ${ }^{2}+$ BY +C

Before attempting to factor these trinomials, make sure you have watched the video on trial and error and the video on grouping to factor trinomials in the form of ax ${ }^{\mathbf{2}}+\mathbf{b y}+\mathbf{c}$. Use whichever method works best for the trinomial. None of these trinomials are prime; they can all be factored.

1. $2 x^{2}+8 x+6$
2. $3 x^{2}+8 x+5$
3. $5 \mathrm{w}^{2}+7 \mathrm{w}+2$
4. $7 \mathrm{a}^{2}+19 a-6$
5. $11 n^{2}+12 x y+y^{2}$
6. $5 \mathrm{a}^{2}-6 \mathrm{ab}+\mathrm{b}^{2}$
7. $4 x^{2}-4 x+1$
(no need for trial/error or grouping)
8. $6 x^{2}+6 x-12$
9. $10 x^{2}+40 x+40$
10. $3 x^{2}-5 x y-2 y^{2}$ College

Although the sum of two squares cannot usually be factored, the sum of two cubes and the difference of two cubes can be factored using the following formulas:

Sum: $\mathbf{a}^{3}+b^{3}=(a+b)\left(a^{2}-a b+b^{2}\right)$
Difference: $\mathbf{a}^{\mathbf{3}}-\mathrm{b}^{\mathbf{3}}=(\mathbf{a}-\mathrm{b})\left(\mathbf{a}^{2}+\mathbf{a b}+\mathrm{b}^{2}\right)$

1. Factor the following sum of cubes and difference of cubes.
a. $\mathrm{x}^{3}+8$
b. $y^{3}+1$
c. $\mathrm{w}^{3}-27$
d. $8 a^{3}+125$
e. $z^{3}-64$
f. $a^{6}+b^{3}$
2. For the following problems, factor out a common factor and then factor the sum or difference of cubes.
a. $125 q^{2}-n^{3} q^{2}$
b. $4 w^{3}+4$
c. $3 x^{6} y^{2}+81 y^{2}$
d. $a x^{3}-a y^{3}$

Adult Learning Academy Elementary Algebra Workbook 6.7 SOLVING QUADRATIC EQUATIONS

Solve the following quadratic equations. Make sure you watch the video on the zero product property and practice solving quadratic equations on Khan Academy before trying these problems.

1. $x^{2}+5 x+6=0$
2. $y^{2}+10=-24$
3. $\mathrm{a}^{2}-49=0$
4. $16 x^{2}+40 x+25=0$
5. $x^{2}=6 x-9$
6. $2 a^{2}-9 a=-7$
7. $4 \mathrm{~b}^{2}-100=0$
8. $a^{2}-6 a=16$
9. $x^{2}-18=-7 x$
10. $\mathrm{a}^{2}=49$

## 1. Factor Completely.

a. $12 \mathrm{a}^{2}+20 \mathrm{a}$
b. $x^{2}-4 x+2 x y-8 y$
c. $x^{2}+17 x+30$
d. $\mathrm{x}^{2}-\mathrm{x}-30$
e. $x^{2}-14 x+30$
f. $x^{2}-25$
g. $x^{2}+49$
h. $3 y^{2}-27$
i. $2 x^{2}+14 x+20$
j. $x^{2}-8 x+16$
k. $7 \mathrm{x}+13 \mathrm{x}+5$

1. $3 x^{3}+9 x^{2}-12 x$
m. $\mathrm{x}^{3}-125$
n. $27 x^{3}+1$
2. Solve the following equations.
a. $x^{2}-24 x-81=0$
b. $2 y^{2}+16 x+30=0$
c. $\mathrm{x}^{2}=100$
d. $x^{2}+14 x=-24$

Adult Learning Academy<br>Elementary Algebra Workbook<br>6.9 CAREER ApPLICATIONS: STEM

1. Find the perimeter of the rectangle and write as a simplified trinomial. Then factor the trinomial completely.


$$
L=x^{2}+2 x
$$

2. Find the perimeter of the triangle and write as a simplified trinomial. Then factor the trinomial completely.

3. The area of a rectangle is $x^{2}+9 x+20$.
a. Find the length and width in terms of $x$. (hint: factor the trinomial)
b. The perimeter of the rectangle is 30 inches. Find the actual dimensions of the rectangle. (hint: solve for x )
c. Find the actual area of the rectangle.
4. The area of a square is 81 units squared. Find the length of a side. s = $\qquad$
5. The area of a square is $\mathbf{x}^{2}+\mathbf{6} \mathbf{x}+\mathbf{9}$.
a. Find the length of a side in terms of x .
b. Find the actual length of a side if the perimeter is 52 units.
c. Find the actual length of a side if the perimeter is 24 units.
d. Find the actual area of the square if the perimeter is 36 units.
e. The trinomial $x^{2}+6 x+9$ is a $\qquad$
$\qquad$ trinomial.
6. An object is dropped from a building that is 841 feet tall. The height of the object after $t$ seconds is 841 - $16 \mathrm{t}^{2}$.
a. Find the height of the object after 2 seconds.
b. Find the height of the object after 5 seconds.
c. To the nearest whole second, estimate when the object hits the ground.
d. Factor $841-16 t^{2}$.
7. An object is dropped from the top of the Woolworth building in New York City. The height h of the object after t seconds is given by the equation $\mathrm{h}=-16 \mathrm{t}^{2}+784$.

Find how many seconds pass before the object reaches the ground.
8. The area of a rectangle is 54 square units. Find the dimensions of the rectangle.

9. Find the lengths of the sides of a right triangle is the hypotenuse is 10 centimeters longer than the shorter leg and 5 centimeters longer than the longer leg. (show all of your work)
10. If the cost, $C$, for manufacturing $x$ units of a certain product is given by $C=x^{2}-15 x+50$, find the number of units manufactured at a cost of $\$ 9500$.
11. Write a quadratic equation in standard form that has two solutions, 6 and -1 .
12. Write a quadratic equation in standard form that has two solutions, 7 and 5.

Module 6 Answer Key
6.1 GCF and Factoring with Distributive Property

1a. 15
1b. $x^{2}$
1c. $2 x^{3}$
1d. $3 x^{3}$
1e. $p^{7} q$
1f. 1

2b. $4(3 x+1)$
2c. 7(6y -1 )
2d. $5 \mathrm{a}\left(3 \mathrm{a}^{2}+1\right)$
2e. $3 x\left(2 x^{2}-3 x+4\right)$
2f. $4(\mathrm{x}-2 \mathrm{y}+1$ )
2g. $x y\left(x y+y^{2}+x^{2}\right)$
2h. $3 q\left(33 q^{6}+8\right)$
2i. $a^{2} b^{2}\left(a^{5} b^{4}+a-b^{3}-1\right)$
3. factors
4. 1, 3, 9, $x, 3 x, 9 x, y, 3 y, 9 y, y^{2}, 3 y^{2}, 9 y^{2}, x y^{2}, 3 x y^{2}, 9 x y^{2}$
5. 1, 7, $\mathbf{y}, 7 \mathrm{y}, \mathrm{y}^{2}, 7 \mathrm{y}^{2}, \mathrm{y}^{3}, 7 \mathrm{y}^{3}$

### 6.2 Factoring by Grouping

1. $x^{2}(x+2)+5(x+2)$

$$
=(x+2)\left(x^{2}+5\right)
$$

2. $x^{2}(x+4)+3(x+4)$ $=(x+4)\left(x^{2}+3\right)$
3. $5(x+3)+y(x+3)$ $=(x+3)(5+y)$
4. $2 x^{2}(3 x-2)+5(3 x-2)$
$=(3 x-2)\left(2 x^{2}+5\right)$
5. $m\left(5 m^{2}+6 n\right)+1\left(5 m^{2}+6 n\right)$
$=\left(5 m^{2}+6 n\right)(m+1)$
6. $2(x-4)+y(x-4)$ $=(x-4)(2+y)$
7. $x(y+3)+3(y+3)$
$=(y+3)(x+3)$
8. $a(b-5)+6(b-5)$
$=(b-5)(a+b)$
6.3 Trinomials in the form of $x^{\underline{2}+b x+c}$

1a. $(x+3)(x+4)$
1b. $(y+9)(y+2)$
1c. $(\mathbf{a}-3)(a+8)$
1d. $(\mathbf{n}-5)(\mathrm{n}-1)$
1e. $(w-10)(w+5)$
1f. Prime
1g. $(b+6)(b+11)$
1h. Prime
1i. $(x+3 y)(x+2 y)$
1j. $\left(x^{2}+3\right)\left(x^{2}+2\right)$
1k. $(\mathrm{t}-8)(\mathrm{t}-5)$
11. $(\mathbf{a}+8)(a-3)$
2. Answers will vary; ex. $x^{2}+7 x+10$
3. Answers will vary; ex. $x^{2}+157 x+10$
4. 9, 12, 21

5a. $2\left(\mathrm{z}^{2}+10 \mathrm{z}+16\right)=2(\mathrm{z}+8)(\mathrm{z}+2)$
5b. $3\left(\mathrm{x}^{2}+10 \mathrm{x}+21\right)=3(\mathrm{x}+7)(\mathrm{x}+3)$
5c. $4\left(x^{2}-x-12\right)=4(x-4)(x+3)$
5d. $x\left(x^{2}+11 x+30\right)=x(x+6)(x+5)$
6. Answers will vary; ex. $100 x^{2}+700 x+1000$

### 6.4 Perfect Sq. Trinomials and Difference of Squares

1a. $(x+10)(x+10)=(x+10)^{2}$
1b. $(\mathbf{a}+1)(\mathrm{a}+1)=(\mathbf{a}+\mathbf{1})^{2}$
1c. $(w-8)(w-8)=(w-8)^{2}$
1d. $(n-3)(n-3)=(n-3)^{2}$
1e. $(x+y)(x+y)=(x+y)^{2}$
1f. $(3 x+1 / 2)(3 x+1 / 2)=(3 x+1 / 2)^{2}$

2a. $(w-9)(w+9)$
2b. $(4 a-1)(4 a+1)$
2c. $(b+3 / 5)(b-3 / 5)$
2d. $\left(c^{2}-d^{3}\right)\left(c^{2}+d^{3}\right)$
2e. $(11 x-1 / 2)(11 x+1 / 2)$
2f. $\left(x^{2} y-z^{4}\right)\left(x^{2} y+z^{4}\right)$

### 6.5 Factoring Trinomials in the Form of ax ${ }^{\underline{2}+\text { by }+\mathbf{c}}$

1. $2\left(x^{2}+4 x+3\right)$

$$
=2(x+3)(x+1)
$$

2. $(3 x+5)(x+1)$
3. $(5 w+2)(w+1)$
4. $(7 a-2)(a+3)$
5. $(11 \mathrm{n}+\mathrm{y})(\mathrm{n}+\mathrm{y})$
6. $(5 a-b)(a-b)$
7. $(2 x-1)(2 x-1)$

$$
=(2 x-1)^{2}
$$

8. $6\left(x^{2}+x-2\right)$

$$
=6(x+2)(x-1)
$$

9. $10\left(x^{2}+4 x+4\right)$

$$
=10(x+2)(x+2)
$$

$$
=10(x+2)^{2}
$$

10. $(3 x+y)(x-2 y)$

### 6.6 Sum and Difference of Cubes

1a. $x^{3}+2^{3}$

$$
=(x+2)\left(x^{2}-4 x+4\right)
$$

1b. $y^{3}+1^{3}$

$$
=(y+1)\left(y^{2}-y+1\right)
$$

1c. $w^{3}-3^{3}$

$$
=(w-3)\left(w^{2}+3 w+9\right)
$$

1d. $(2 \mathrm{a})^{3}+5^{3}$

$$
=(2 a+5)\left(4 a^{2}-10 a+25\right)
$$

1e. $z^{3}-4^{3}$

$$
=(z-4)\left(z^{2}-4 z+16\right)
$$

1f. $\left(a^{2}\right)^{3}+b^{3}$

$$
=\left(a^{2}+b\right)\left(a^{4}-a^{2} b+b^{2}\right)
$$

2a. $q^{2}\left(125-n^{3}\right)$

$$
\begin{aligned}
& =q^{2}\left(5^{3}-n^{3}\right) \\
& =\mathbf{q}^{2}(5-\mathbf{n})\left(25+5 \mathbf{n}+\mathbf{n}^{2}\right)
\end{aligned}
$$

2b. $4\left(w^{3}+1\right)$

$$
=4\left(w^{3}+1^{3}\right)
$$

$$
4(w+1)\left(w^{2}-w+1\right)
$$

2c. $3 y^{2}\left(x^{6}+27\right)$

$$
\begin{aligned}
& =3 y^{2}\left[\left(x^{2}\right)^{3}+3^{3}\right] \\
& =3 y^{2}\left(\mathbf{x}^{2}+3\right)\left(x^{4}-3 x^{2}+9\right)
\end{aligned}
$$

2d. $a\left(x^{3}-y^{3}\right)$

$$
=a(x-y)\left(x^{2}+x y+y^{2}\right)
$$

### 6.7 Solving Quadratic Equations

1. $(x+3)(x+2)=0$
$x+3=0$ or $x+2=0$
$x=-3$ or $x=-2$
2. $y^{2}+10 y+24=0$
$(y+6)(y+4)=0$
$y+6=0$ or $y+4=0$
$y=-6$ or $y=-4$
3. $(a-7)(a+7)=0$
$a-7=0$ or $a+7=0$
$\mathbf{a}=7$ or $\mathbf{a}=-7$
4. $(4 x+5)(4 x+5)=0$
$4 \mathrm{x}+5=0$ or $4 \mathrm{x}+5=0$
$x=-5 / 4$
5. $x^{2}-6 x+9=0$
$(x-3)(x-3)=0$
$x-3=0$
$\mathrm{x}=3$
6. $2 \mathrm{a}^{2}-9 \mathrm{a}+7=0$
$(2 a-7)(a-1)=0$
$2 \mathrm{a}-7=0$ or $\mathrm{a}-1=0$
$a=7 / 2$ or $\mathbf{a}=1$
7. $4\left(\mathrm{~b}^{2}-25\right)=0$
$4(b-5)(b+5)=0$
$\mathrm{b}-5=0$ or $\mathrm{b}+5=0$
$b=5$ or $b=-5$
8. $a^{2}-6 a-16=0$
$(a-8)(a+2)=0$
$\mathrm{a}-8=0$ or $\mathrm{a}+2=0$
$\mathbf{a}=8$ or $\mathbf{a}=-2$
9. $x^{2}+7 x-18=0$
$(x+9)(x-2)=0$
$\mathrm{x}+9=0$ or $\mathrm{x}-2=0$
$x=-9$ or $x=2$
10. $\mathrm{a}^{2}-49=0$
$(a-7)(a+7)=0$
$\mathrm{a}-7=0$ or $\mathrm{a}+7=0$
$\mathbf{a}=7$ or $\mathbf{a}=-7$

### 6.8 Factoring Review

1a. $4 a(3 a+5)$
1b. $x(x-4)+2 y(x-4)$ $=(x+2 y)(x-4)$
1c. $(x+15)(x+2)$
1d. $(x-6)(x+5)$
1e. Prime
1f. $(x+5)(x-5)$
1g. Prime

### 6.8 Factoring Review (cont.)

1h. $3\left(y^{2}-9\right)$

$$
=3(y-3)(y+3)
$$

1h. $3\left(y^{2}-9\right)$

$$
=3(y-3)(y+3)
$$

1i. $2\left(x^{2}+7 x+10\right)$
$=2(x+5)(x+2)$
1i. $(x-4)(x-4)$
$=(x-4)^{2}$
1j. Prime
1k. $3 x\left(x^{2}+3-4\right)$

$$
=3 x(x+4)(x-1)
$$

1k. $a=x, b=5$

$$
=(x-5)\left(x^{2}+5 x+25\right)
$$

1k. $a=3 x, b=1$

$$
=(3 x+1)\left(9 x^{2}-3 x+1\right)
$$

2a. $(x-27)(x+3)=0$

$$
\begin{aligned}
& x-27=0 \text { or } x+3=0 \\
& x=27 \text { or } x=-3
\end{aligned}
$$

2b. $2\left(y^{2}+8 x+15\right)=0$
$2(y+5)(y+3)=0$
$\mathrm{y}+5=0$ or $\mathrm{y}+3=0$
$y=-5$ or $y=-3$
2c. $x=10$ or $x=-10$
2d. $x^{2}+14 x+24=0$

$$
\begin{aligned}
& (x+12)(x+2)=0 \\
& x+12=0 \text { or } x+2=0 \\
& y=-12 \text { or } y=-2
\end{aligned}
$$

### 6.9 Career Applications: STEM

1. To find perimeter, add all sides:
$x^{2}+2 x+x^{2}+2 x+5 x+10+5 x+10$
$=2 \mathrm{x}^{2}+14 \mathrm{x}+20$
$=2\left(x^{2}+7 x+10\right)$
$=2(x+5)(x+2)$
2. $x^{2}+8+x^{2}-9 x+x^{2}-20$
$=3 x^{2}-9 x-12$
$=3\left(x^{2}-3 x-4\right)$
$=3(x-4)(x+1)$
3a. $(x+5)(x+4)$
3b. $x+5+x+5+x+4+x+4=30$
$4 x+18=30$
$4 \mathrm{x}=12$
$\mathrm{x}=3$
length $=3+5=8 \mathrm{in}$.
width $=3+4=7$ in.
3c. To find area, multiply length by width: $\mathbf{8 \times 7 = 5 6} \mathbf{i n}$.

### 6.9 Career Applications: STEM (cont.)

4. $\mathrm{s}^{2}=81$
$\mathrm{s}=\sqrt{81}$
$\mathrm{s}=\mathbf{9}$

5a. $x+3$
5b. $4(x+3)=52$
$4 x+12=52$
$4 \mathrm{x}=40$
$\mathrm{x}=10$
5c. $4(\mathrm{x}+3)=24$
$4 x+12=24$
$4 \mathrm{x}=12$
$\mathrm{x}=3$
5d. area $=36=6^{2}$ so each side is 6
$6+6+6+6=\mathbf{2 4}$
5e. perfect square
6a. $t=2$
$h=841-16(4)=841-64$
$h=777$ feet high
6b. $t=5$

$$
\begin{aligned}
& \mathrm{h}=841-16(25)=841-400 \\
& \mathbf{h}=441 \text { feet high }
\end{aligned}
$$

6c. $\mathrm{h}=0$ (height of ground)
To estimate, try different values of $t$.
If $\mathrm{t}=7, \mathrm{~h}=841-16(49)=841-784=57$
$\mathbf{t}=7$ is the closest you can get to the ground in
whole numbers of seconds
To solve exactly
$0=841-16 \mathrm{t}^{2}$
$0=29^{2}-16 t^{2}$
$0=(29+4 \mathrm{t})(29-4 \mathrm{t})$
$29+4 \mathrm{t}=0$ or $29-4 \mathrm{t}=0$
$\mathrm{t}=29 / 4$ or $\mathrm{t}=-29 / 4$
$\mathbf{t}=7.25$ (negative time does not make sense)
6d. $(29-4 t)(29+4 t)$
7. $0=-16 t^{2}+784$
$0=-16\left(\mathrm{t}^{2}-49\right)$
$0=-16(\mathrm{t}-7)(\mathrm{t}+7)$
$\mathrm{t}-7=0$ or $\mathrm{t}+7=0$
t = 7 seconds ( -7 does not make sense)

### 6.9 Career Applications: STEM (cont.)

8. $(x-1)(x+1)=54$
$x^{2}+x-2=54$
$x^{2}+x-56=0$
$(x+8)(x-7)=0$
$\mathrm{x}+8=0$ or $\mathrm{x}-7=0$
$\mathbf{x}=7$ (-8 does not make sense)
length $=7+2=9$
width $=7-1=6$
9. Pythagorean Theorem: $a^{2}+b^{2}=c^{2}$
let $x=$ shorter leg, hypotenuse $=x+10$, longer leg $=x$
$+5$
$x^{2}+(x+5)^{2}=(x+10)^{2}$
$x^{2}+x^{2}+10 x+25=x^{2}+20 x+100$
$\mathrm{x}^{2}-10 \mathrm{x}-75=0$
$(x-15)(x+5)=0$
$x-15=0$ or $x+5=0$
$x=15$ or $x=-5$
10. $9500=x^{2}-15 x+50$
$x^{2}-15 x-9450=0$
$(x+90)(x-105)=0$
$x+90=0$ or $x-105=0$
$\mathbf{x}=105$ (-90 does not make sense)
11. $(x-6)(x+1)=0$
$x^{2}-5 x-6=0$
12. $(x-7)(x-5)=0$
$x^{2}-12 x+35=0$


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