St. Louis Community College

Adult Learning Academy
Elementary Algebra Workbook

MoSTEM WINS

MODULE 6: FACTORING POLYNOMIALS, SOLVING QUADRATIC EQUATIONS BY FACTORING

LEARNING OBJECTIVES

By the time you finish this module, you should be able to:

- ☐ Factor polynomials, choosing from a variety of strategies:
  - o Greatest Common Factor
  - o Reverse Foil
  - o Difference of Two Squares
  - o Factoring by Grouping
  - o Difference of Cubes
  - o Sum of Cubes
- ☐ Recognize Prime polynomials
- ☐ Solve quadratic equations by factoring
- ☐ Create quadratic equations to model situations and solve application problems

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IMPORTANT INFORMATION FROM MODULE 6:

$a^2 - b^2 = (a - b)(a + b)$

$a^2 + b^2$ *is prime*

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

ALWAYS check for COMMON FACTORS!
If a polynomial doesn't factor, then it's prime.
Always check inside parentheses to be sure you have factored completely.
Check your factoring by multiplying your factors back to get the original polynomial.

A quadratic equation contains an x2. It has up to two solutions. The solutions are also the x-intercepts of the parabola if you graph the equation. To solve a quadratic equation by factoring:

-- Get 0 on one side of the equation
-- Factor the other side
-- Set each factor = to 0
-- Solve

Topic	Website	Videos	Exercises
Factoring with the	www.khanacademy.org	Factoring/distributive property 1	(refer to workbook)
distributive property		Factoring/distributive property 2	
		Factoring/distributive property 3	
		Factoring by grouping	
Factoring Quadratic	www.khanacademy.org	Factoring a Quadratic Expression	Factoring Quadratic Expressions
Expressions		Factoring a Quadratic Expression 2	More factoring trinomials
	youtube.com	Recognizing a Prime trinomial	
Factoring Special	www.khanacademy.org	Factoring Difference of Squares	Factoring Difference of Squares
Products		Factoring Difference of Squares 2	More Factoring Diff. of Squares
		Factoring Perfect Square Trinomials	Factoring Perfect Square Trinomials
	youtube.com	Factoring Rap	
Factoring trinomials in	youtube.com	Factoring by trial and error	Factoring by grouping
the form $ax^2 + by + c$	www.khanacademy.org	Factoring by grouping (3 videos)	
	youtube.com	Factoring Completely	
Factoring Cubes	Khanacademy.org	Difference of Cubes	Refer to workbook
		Sum of Cubes	
		Difference and Sum of Cubes	
Quadratic Equations	youtube.com	Zero Property Rule	Solving quadratic equations by factoring
	Khanacademy.org	Solving Quadratic Equations by factoring	
Module 6 Test Review	stlcc.edu	Blackboard PowerPoint	

6.1 Greatest Common Factor and Factoring Using the Distributive Property

1. Find the GCF from the list of terms.

   a. 30, 75, 135 _____

   b. x^3, x^2, x^5 _____

   c. $32x^5$, $18x^2$ _____

   d. $12x^3$, $6x^4$, $3x^5$ _____

   e. p^7q, p^8q^2, p^9q^3 _____

   f. x^3, y^3, z^3 _____

2. Factor out the GCF from each polynomial. The first one is done for you.

   a. $3a + 6$ $3(a + 2)$

   b. $12x + 4$ _____

   c. $42y - 7$ _____

   d. $15a^3 + 5a$ _____

   e. $6x^3 - 9x^2 + 12x$ _____

   f. $4x - 8y + 4$ _____

   g. $^2y^2 + xy^3 + x^3y$ _____

   h. $99q^7 + 24q$ _____

   i. $a^7b^6 + a^3b^2 - a^2b^5 - a^2b^2$ _____

3. Since $5 \cdot 4 = 20$, the numbers 5 and 4 are called _____ of 20.

4. List all the factors of $9xy^2$ (hint: there are 14 factors)

   _____

5. List all the factors of $7y^3$

   _____

To factor by grouping:

1. Group the terms in two groups so that each group has a common factor.
2. Factor out the GCF from each group.
3. If there is a common binomial factor, factor it out.
4. If not, rearrange the terms and try again.

Example: Factor $xy + 2x + 3y + 6$

1. Group terms: $(xy + 2x) + (3y + 6)$
2. Factor out the GCF: $x(y + 2) + 3(y + 2)$
3. Factor out the binomial: **$(y + 2)(x + 3)$**

Factor by grouping:

1. $x^3 + 2x^2 + 5x + 10$
2. $x^3 + 4x^2 + 3x + 12$
3. $5x + 15 + xy + 3y$
4. $6x^3 - 4x^2 + 15x - 10$
5. $5m^3 + 6mn + 5m^2 + 6n$
6. $2y - 8 + xy - 4y$
7. $xy + 3x + 3y + 9$
8. $ab - 5a + 6b -$

1. Factor each trinomial. If the trinomial cannot be factored, write prime. Check your answer by using foil.

   a. $x^2 + 7x + 12$ b. $y^2 + 11y + 18$ c. $a^2 + 5a - 24$

   d. $n^2 - 6n + 5$ e. $w^2 - 5w - 50$ f. $a^2 + 3a + 11$

   g. $b^2 + 17b + 66$ h. $x^2 - x - 10$ i. $x^2 + 5xy + 6y^2$

   j. $x^4 + 5x^2 + 6$ k. k. $40 - 13t + t^2$ l. $-24 + 5a + a^2$

2. Write a trinomial that is not prime: _____

3. Write a trinomial that is prime: _____

4. Find all positive values of b so that the trinomial is factorable: $y^2 + by + 20$

5. Factor each trinomial completely. Factor out the GCF first.

   a. $2z^2 + 20z + 32$ b. $3x^2 + 30x\ 63$

   c. $4x^2 - 4x - 48$ d. $x^3 + 11x^2 + 30x$

6. Write a trinomial that must have its GCF factored out before you can factor.

   _____

A **perfect square trinomial** is a trinomial that is the square of a binomial.

For example: $(x+5)^2 = (x+5)(x+5) = x^2 + 10x + 25$

The following formulas can be used when factoring perfect square trinomials:

$$\mathbf{a^2 + 2ab + b^2 = (a + b)^2} \quad \text{and} \quad \mathbf{a^2 - 2ab + b^2 = (a - b)^2}$$

1. Factor the following perfect square trinomials. Write your answer with an exponent.

   a. $x^2 + 20x + 100$

   b. $a^2 + 2a + 1$

   c. $w^2 - 16w + 64$

   d. $n^2 - 6n + 9$

   e. $x^2 + 2xy + y^2$

   f. $9x^2 + 3x + \frac{1}{4}$

The **difference of two squares** is another type of special product. Consider the difference between these two perfect squares: $w^2 - 49 = (w+7)(w-7)$ Notice when foil is used to multiply the two binomials, the middle term cancels out.

The following formula can be used to factor the difference of squares:

$$\mathbf{a^2 - b^2 = (a+b)(a - b)}$$

2. Factor the following difference of squares.

   a. $w^2 - 81$

   b. $16a^2 - 1$

   c. $b^2 - \frac{9}{25}$

   d. $c^4 - d^6$

   e. $121x^2 - \frac{1}{4}$

   f. $x^4y^2 - z^8$

Before attempting to factor these trinomials, make sure you have watched the video on **trial and error** and the video on **grouping to factor trinomials in the form of $ax^2 + by + c$**. Use whichever method works best for the trinomial. None of these trinomials are prime; they can all be factored.

1. $2x^2 + 8x + 6$

2. $3x^2 + 8x + 5$

3. $5w^2 + 7w + 2$

4. $7a^2 + 19a - 6$

5. $11n^2 + 12xy + y^2$

6. $5a^2 - 6ab + b^2$

7. $4x^2 - 4x + 1$
   (no need for trial/error or grouping)

8. $6x^2 + 6x - 12$

9. $10x^2 + 40x + 40$

10. $3x^2 - 5xy - 2y^2$

Although the sum of two squares cannot usually be factored, the sum of two cubes and the difference of two cubes can be factored using the following formulas:

Sum: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Difference: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

1. Factor the following sum of cubes and difference of cubes.

   a. $x^3 + 8$

   b. $y^3 + 1$

   c. $w^3 - 27$

   d. $8a^3 + 125$

   e. $z^3 - 64$

   f. $a^6 + b^3$

2. For the following problems, factor out a common factor and then factor the sum or difference of cubes.

   a. $125q^2 - n^3q^2$

   b. $4w^3 + 4$

   c. $3x^6y^2 + 81y^2$

   d. $ax^3 - ay^3$

Solve the following quadratic equations. Make sure you watch the video on the **zero product property** and practice solving quadratic equations on Khan Academy before trying these problems.

1. $x^2 + 5x + 6 = 0$
2. $y^2 + 10 = -24$
3. $a^2 - 49 = 0$
4. $16x^2 + 40x + 25 = 0$
5. $x^2 = 6x - 9$
6. $2a^2 - 9a = -7$
7. $4b^2 - 100 = 0$
8. $a^2 - 6a = 16$
9. $x^2 - 18 = -7x$
10. $a^2 = 49$

1. Factor Completely.

   a. $12a^2 + 20a$

   b. $x^2 - 4x + 2xy - 8y$

   c. $x^2 + 17x + 30$

   d. $x^2 - x - 30$

   e. $x^2 - 14x + 30$

   f. $x^2 - 25$

   g. $x^2 + 49$

   h. $3y^2 - 27$

   i. $2x^2 + 14x + 20$

   j. $x^2 - 8x + 16$

   k. $7x + 13x + 5$

   l. $3x^3 + 9x^2 - 12x$

   m. $x^3 - 125$

   n. $27x^3 + 1$

2. Solve the following equations.

   a. $x^2 - 24x - 81 = 0$

   b. $2y^2 + 16x + 30 = 0$

   c. $x^2 = 100$

   d. $x^2 + 14x = -24$

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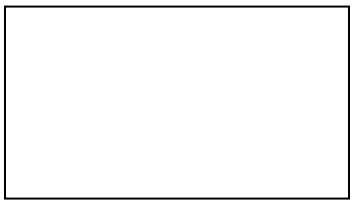
6.9 CAREER APPLICATIONS: STEM

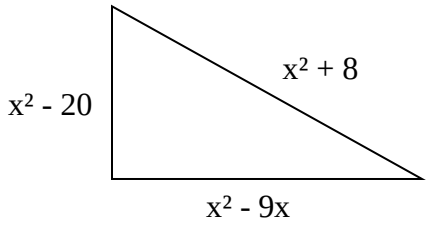
1. Find the perimeter of the rectangle and write as a simplified trinomial. Then factor the trinomial completely.



W = 5x + 10

L = $x^2 + 2x$

2. Find the perimeter of the triangle and write as a simplified trinomial. Then factor the trinomial completely.



3. The area of a rectangle is $x^2 + 9x + 20$.

   a. Find the length and width in terms of x. (hint: factor the trinomial)

   b. The perimeter of the rectangle is 30 inches. Find the actual dimensions of the rectangle. (hint: solve for x)

   c. Find the actual area of the rectangle.

4. The area of a square is 81 units squared. Find the length of a side. s = _____

5. The area of a square is $\mathbf{x^2 + 6x + 9}$.

   a. Find the length of a side in terms of x.

   b. Find the actual length of a side if the perimeter is 52 units.

   c. Find the actual length of a side if the perimeter is 24 units.

   d. Find the actual **area** of the square if the perimeter is 36 units.

   e. The trinomial $x^2 + 6x + 9$ is a _____ _____ trinomial.

6. An object is dropped from a building that is 841 feet tall. The height of the object after t seconds is $841 – 16t^2$.

   a. Find the height of the object after 2 seconds.

   b. Find the height of the object after 5 seconds.

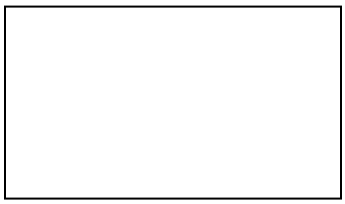
   c. To the nearest whole second, estimate when the object hits the ground.

   d. Factor $841 – 16t^2$.

7. An object is dropped from the top of the Woolworth building in New York City. The height h of the object after t seconds is given by the equation $h = -16t^2 + 784$.

   Find how many seconds pass before the object reaches the ground.

8. The area of a rectangle is 54 square units. Find the dimensions of the rectangle.



W = x - 1

L = x + 2

9. Find the lengths of the sides of a right triangle is the hypotenuse is 10 centimeters longer than the shorter leg and 5 centimeters longer than the longer leg. (show all of your work)

10. If the cost, C, for manufacturing x units of a certain product is given by $C = x^2 - 15x + 50$, find the number of units manufactured at a cost of $9500.

11. Write a quadratic equation in standard form that has two solutions, 6 and – 1.

12. Write a quadratic equation in standard form that has two solutions, 7 and 5.

6.1 GCF and Factoring with Distributive Property

1a. 15
1b. x^2
1c. $2x^3$
1d. $3x^3$
1e. p^7q
1f. 1

2b. $4(3x + 1)$
2c. $7(6y – 1)$
2d. $5a(3a^2 + 1)$
2e. $3x(2x^2 – 3x + 4)$
2f. $4(x – 2y + 1)$
2g. $xy(xy + y^2 + x^2)$
2h. $3q(33q^6 + 8)$
2i. $a^2b^2(a^5b^4 + a – b^3 – 1)$

3. factors
4. $1, 3, 9, x, 3x, 9x, y, 3y, 9y, y^2, 3y^2, 9y^2, xy^2, 3xy^2, 9xy^2$
5. $1, 7, y, 7y, y^2, 7y^2, y^3, 7y^3$

6.2 Factoring by Grouping

1. $x^2(x + 2) + 5(x + 2)$
$= (x + 2)(x^2 + 5)$

2. $x^2(x + 4) + 3(x + 4)$
$= (x + 4)(x^2 + 3)$

3. $5(x + 3) + y(x + 3)$
$= (x + 3)(5 + y)$

4. $2x^2(3x – 2) + 5(3x – 2)$
$= (3x – 2)(2x^2 + 5)$

5. $m(5m^2 + 6n) + 1(5m^2 + 6n)$
$= (5m^2 + 6n)(m + 1)$

6. $2(x – 4) + y(x – 4)$
$= (x – 4)(2 + y)$

7. $x(y + 3) + 3(y + 3)$
$= (y + 3)(x + 3)$

8. $a(b – 5) + 6(b – 5)$
$= (b – 5)(a + b)$

6.3 Trinomials in the form of $x^2 + bx + c$

1a. $(x + 3)(x + 4)$
1b. $(y + 9)(y + 2)$
1c. $(a – 3)(a + 8)$
1d. $(n – 5)(n – 1)$
1e. $(w – 10)(w + 5)$
1f. Prime
1g. $(b + 6)(b + 11)$
1h. Prime
1i. $(x + 3y)(x + 2y)$
1j. $(x^2 + 3)(x^2 + 2)$
1k. $(t – 8)(t – 5)$
1l. $(a+ 8)(a – 3)$

2. Answers will vary; ex. $x^2 + 7x + 10$
3. Answers will vary; ex. $x^2 + 157x + 10$
4. 9, 12, 21

5a. $2(z^2 + 10z + 16) = 2(z + 8)(z + 2)$
5b. $3(x^2 + 10x + 21) = 3(x + 7)(x + 3)$
5c. $4(x^2 – x – 12) = 4(x – 4)(x + 3)$
5d. $x(x^2 + 11x + 30) = x(x + 6)(x + 5)$

6. Answers will vary; ex. $100x^2 + 700x + 1000$

6.4 Perfect Sq. Trinomials and Difference of Squares

1a. $(x + 10)(x + 10) = (x + 10)^2$
1b. $(a + 1)(a + 1) = (a + 1)^2$
1c. $(w – 8)(w – 8) = (w – 8)^2$
1d. $(n – 3)(n – 3) = (n – 3)^2$
1e. $(x + y)(x + y) = (x + y)^2$
1f. $(3x + ½)(3x + ½) = (3x + ½)^2$

2a. $(w – 9)(w + 9)$
2b. $(4a – 1)(4a + 1)$
2c. $(b + 3/5)(b – 3/5)$
2d. $(c^2 – d^3)(c^2 + d^3)$
2e. $(11x – ½)(11x + ½)$
2f. $(x^2y – z^4)(x^2y + z^4)$

6.5 Factoring Trinomials in the Form of $ax^2 + by + c$

1. $2(x^2 + 4x + 3)$
$= 2(x + 3)(x + 1)$

2. **$(3x + 5)(x + 1)$**

3. **$(5w + 2)(w + 1)$**

4. **$(7a - 2)(a + 3)$**

5. **$(11n + y)(n + y)$**

6. **$(5a – b)(a – b)$**

7. $(2x – 1)(2x – 1)$
$= (2x – 1)^2$

8. $6(x^2 + x – 2)$
$= 6(x + 2)(x – 1)$

9. $10(x^2 + 4x + 4)$
$= 10(x + 2)(x + 2)$
$= 10(x + 2)^2$

10. **$(3x + y)(x – 2y)$**

6.6 Sum and Difference of Cubes

1a. $x^3 + 2^3$
$= (x + 2)(x^2 - 4x + 4)$

1b. $y^3 + 1^3$
$= (y + 1)(y^2 - y + 1)$

1c. $w^3 - 3^3$
$= (w – 3)(w^2 + 3w + 9)$

1d. $(2a)^3 + 5^3$
$= (2a + 5)(4a^2 – 10a + 25)$

1e. $z^3 – 4^3$
$= (z – 4)(z^2 – 4z + 16)$

1f. $(a^2)^3 + b^3$
$= (a^2 + b)(a^4 – a^2b + b^2)$

2a. $q^2(125 – n^3)$
$= q^2(5^3 – n^3)$
$= q^2(5 – n)(25 + 5n + n^2)$

2b. $4(w^3 + 1)$
$= 4(w^3 + 1^3)$
$4(w + 1)(w^2 – w + 1)$

2c. $3y^2(x^6 + 27)$
$= 3y^2[(x^2)^3 + 3^3]$
$= 3y^2(x^2 + 3)(x^4 - 3x^2 + 9)$

2d. $a(x^3 – y^3)$
$= a(x – y)(x^2 + xy + y^2)$

6.7 Solving Quadratic Equations

1. $(x + 3)(x + 2) = 0$
$x + 3 = 0$ or $x + 2 = 0$
$x = -3$ or $x = -2$

2. $y^2 + 10y + 24 = 0$
$(y + 6)(y + 4) = 0$
$y + 6 = 0$ or $y + 4 = 0$
$y = -6$ or $y = -4$

3. $(a – 7)(a + 7) = 0$
$a – 7 = 0$ or $a + 7 = 0$
$a = 7$ or $a = -7$

4. $(4x + 5)(4x + 5) = 0$
$4x + 5 = 0$ or $4x + 5 = 0$
$x = -5/4$

5. $x^2 – 6x + 9 = 0$
$(x – 3)(x – 3) = 0$
$x – 3 = 0$
$x = 3$

6. $2a^2 – 9a + 7 = 0$
$(2a – 7)(a – 1) = 0$
$2a – 7 = 0$ or $a – 1 = 0$
$a = 7/2$ or $a = 1$

7. $4(b^2 – 25) = 0$
$4(b – 5)(b + 5) = 0$
$b – 5 = 0$ or $b + 5 = 0$
$b = 5$ or $b = -5$

8. $a^2 – 6a – 16 = 0$
$(a – 8)(a + 2) = 0$
$a – 8 = 0$ or $a + 2 = 0$
$a = 8$ or $a = -2$

9. $x^2 + 7x – 18 = 0$
$(x + 9)(x – 2) = 0$
$x + 9 = 0$ or $x – 2 = 0$
$x = -9$ or $x = 2$

10. $a^2 – 49 = 0$
$(a – 7)(a + 7) = 0$
$a – 7 = 0$ or $a + 7 = 0$
$a = 7$ or $a = -7$

6.8 Factoring Review

1a. **$4a(3a + 5)$**

1b. $x(x-4) + 2y(x-4)$
$= (x+2y)(x-4)$

1c. **$(x + 15)(x + 2)$**

1d. **$(x – 6)(x + 5)$**

1e. **Prime**

1f. **$(x + 5)(x – 5)$**

1g. **Prime**

6.8 Factoring Review (cont.)

1h. $3(y^2 - 9)$
$= 3(y - 3)(y + 3)$

1h. $3(y^2 - 9)$
$= 3(y - 3)(y + 3)$

1i. $2(x^2 + 7x + 10)$
$= 2(x + 5)(x + 2)$

1i. $(x - 4)(x - 4)$
$= (x - 4)^2$

1j. Prime

1k. $3x(x^2 + 3 - 4)$
$= 3x(x + 4)(x - 1)$

1k. *a = x, b = 5*
$= (x-5)(x^2 + 5x + 25)$

1k. *a = 3x, b = 1*
$= (3x + 1)(9x^2 - 3x + 1)$

2a. $(x - 27)(x + 3) = 0$
$x - 27 = 0$ or $x + 3 = 0$
$x = 27$ or $x = -3$

2b. $2(y^2 + 8x + 15) = 0$
$2(y + 5)(y + 3) = 0$
$y + 5 = 0$ or $y + 3 = 0$
$y = -5$ or $y = -3$

2c. $x = 10$ or $x = -10$

2d. $x^2 + 14x + 24 = 0$
$(x + 12)(x + 2) = 0$
$x + 12 = 0$ or $x + 2 = 0$
$y = -12$ or $y = -2$

6.9 Career Applications: STEM

1. *To find perimeter, add all sides:*

$x^2 + 2x + x^2 + 2x + 5x + 10 + 5x + 10$

$= 2x^2 + 14x + 20$

$= 2(x^2 + 7x + 10)$

$= 2(x + 5)(x + 2)$

2. $x^2 + 8 + x^2 - 9x + x^2 - 20$

$= 3x^2 - 9x - 12$

$= 3(x^2 - 3x - 4)$

$= 3(x - 4)(x + 1)$

3a. $(x + 5)(x + 4)$

3b. $x + 5 + x + 5 + x + 4 + x + 4 = 30$
$4x + 18 = 30$
$4x = 12$
$x = 3$
length = 3 + 5 = 8 in.
width = 3 + 4 = 7 in.

3c. *To find area, multiply length by width:* **8 x 7 = 56 in.**

6.9 Career Applications: STEM (cont.)

4. $s^2 = 81$
$s = \sqrt{81}$
$s = 9$

5a. $x + 3$

5b. $4(x + 3) = 52$
$4x + 12 = 52$
$4x = 40$
$x = 10$

5c. $4(x + 3) = 24$
$4x + 12 = 24$
$4x = 12$
$x = 3$

5d. area $= 36 = 6^2$ so each side is 6
$6 + 6 + 6 + 6$ **$= 24$**

5e. perfect square

6a. $t = 2$
$h = 841 - 16(4) = 841 - 64$
h = 777 feet high

6b. $t = 5$
$h = 841 - 16(25) = 841 - 400$
h = 441 feet high

6c. $h = 0$ (height of ground)
To estimate, try different values of t.
If $t = 7$, $h = 841 - 16(49) = 841 - 784 = 57$
t = 7 is the closest you can get to the ground in whole numbers of seconds

To solve exactly
$0 = 841 - 16t^2$
$0 = 29^2 - 16t^2$
$0 = (29 + 4t)(29 - 4t)$
$29 + 4t = 0$ or $29 - 4t = 0$
$t = 29/4$ or $t = -29/4$
t = 7.25 (negative time does not make sense)

6d. $(29 - 4t)(29 + 4t)$

7. $0 = -16t^2 + 784$
$0 = -16(t^2 - 49)$
$0 = -16(t - 7)(t + 7)$
$t - 7 = 0$ or $t + 7 = 0$
t = 7 seconds (-7 does not make sense)

6.9 Career Applications: STEM (cont.)

8. $(x - 1)(x + 1) = 54$
$x^2 + x - 2 = 54$
$x^2 + x - 56 = 0$
$(x + 8)(x - 7) = 0$
$x + 8 = 0$ or $x - 7 = 0$
x = 7 (-8 does not make sense)
length = 7 + 2 = 9
width = 7 – 1 = 6

9. *Pythagorean Theorem:* $a^2 + b^2 = c^2$
let x = shorter leg, hypotenuse = x + 10, longer leg = x + 5
$x^2 + (x + 5)^2 = (x + 10)^2$
$x^2 + x^2 + 10x + 25 = x^2 + 20x + 100$
$x^2 - 10x - 75 = 0$
$(x - 15)(x + 5) = 0$
$x - 15 = 0$ or $x + 5 = 0$
x = 15 or x = -5

10. $9500 = x^2 - 15x + 50$
$x^2 - 15x - 9450 = 0$
$(x + 90)(x - 105) = 0$
$x + 90 = 0$ or $x - 105 = 0$
x = 105 (-90 does not make sense)

11. $(x - 6)(x + 1) = 0$
x^2 -5x – 6 = 0

12. $(x - 7)(x - 5) = 0$
x^2 – 12x + 35 = 0

MoSTEMWINs

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