

### Adult Learning Academy Elementary Algebra Workbook



# MODULE 5: INTEGER EXPONENTS & LAWS, SCIENTIFIC NOTATION, POLYNOMIALS & OPERATIONS

#### **LEARNING OBJECTIVES**

By the time you finish this module, you should be able to:

- □ Simplify expressions involving positive or negative exponents according to the exponent rules
- □ Convert a number in standard notation into scientific notation
- □ Convert a number in scientific notation into standard notation
- □ Identify a monomial, binomial, trinomial, and polynomial
- □ Sort polynomials according to their degree
- □ Add, subtract, and multiply polynomials of any size
- $\Box$  Divide a polynomial by a monomial

#### **IMPORTANT INFORMATION FROM MODULE 5:**

 $x^m \cdot x^n = x^{m+n}$ 

 $(x^m)^n = x^{mn}$  "when you raise a power to a power, mutiply the powers"

$$\frac{x^m}{x^n} = x^{m-n}$$

 $x^0 = 1$  ,  $0^m = 0$  ,  $0^0$  is undefined

 $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$   $x^{-m} = \frac{1}{x^m}$  A negative exponent does NOT make a number negative!

one term: monomial; two terms: binomial; three terms: trinomial

To find the degree of a single term, add all the exponents on all the variables in that term. To find the degree of a polynomial, have a contest: the term with the highest degree wins!

FOIL: for multiplying a binomial times a binomial (also for squaring a binomial!!)  $(a+b)^2 = a^2 + 2ab + b^2$   $(a-b)^2 = a^2 - 2ab + b^2$   $(a+b)(a-b) = a^2 - b^2$ 

Birthday Song: You must have like terms, you must have like terms — to ADD or SUBTRACT, you must have like terms!

When you subtract a polynomial, be sure to subtract EVERY term!!



#### Adult Learning Academy Elementary Algebra Workbook MODULE 5 VIDEO & EXERCISE LIST



Торіс	Website	Videos	Exercises
Exponents	www.khanacademy.org	Level 1 Exponents	Positive and Zero Exp
		Understanding Exponents 2	Negative Exponents
		Understanding Exponents	Exponent Rules
		Level 2 Exponents (negative exp)	
		Exponent Rules Part 1	
		Exponent Rules Part 2	
		Exponent Prop involving Quotients	
	www.stlcc.edu	Exponent Rules ppt on Blackboard	
	http://www.youtube.com/watch?v=h063AzwjGlc	Mathman: 3 Exponent Mistakes	
Scientific Notation	www.khanacademy.org	Scientific Notation	Scientific Notation
		Scientific Notation 1	
Polynomials	http://www.youtube.com/watch?v=D-3NIysYshM	Diff betw Trinom, Bi, Monomial	
	http://www.youtube.com/watch?v=l_kY3sHViSA	Identifying Degree, Name of Polyn.	
	www.khanacademy.org	Tems Coefficients and Exponents	
		Evaluating a Polynomial at a Given Value	
		Simplify a Polynomial	
Add, Subt. Polynom.	www.khanacademy.org	Adding Polynomials	Adding, Sub Polynom.
		Ex: Adding Polynomials w/Mult Var.	
		Add & Subt of Polynomials	
		Adding and Sub Polynomials 1	
		Adding & Subt Polynomials 2	
		Adding and Sub Polynomials 3	
		Subtracting Polynomials	
		Sub Polynomials w/ Mult Variables	

Торіс	Website	Videos	Exercises
Multiplying Polynom.	www.khanacademy.org	Multiplying Monomials	Multip. Express. 0.5
		Multiplying Monomials by Polynom.	Multiplying Exp. 1
		Multiplying Binomials	Multiplying Polynom.
		Multiplying Polynomials1	
		Multiplication of Polynomials	
		Square a Binomial	
		Special Products of Binomials	
		Special Polynomials Products 1	
		Special Products of Polynomials 1	
		Special Products of Polynomials 2	
		Multiplying Polynomials	
		More Multiplying Polynomials	
Dividing Polynomials	www.khanacademy.org	Polynomial Divided by Monomial	
		Dividing Multivariable Poly. w/ Mono	
Module 5 Test Review	www.stlcc.edu	Blackboard PowerPoint	Exponent Rules
			Module 5 Review Flashcards

MoSTEMWINs

This product is 100% funded by the MoSTEMWINS \$19.7 million grant from the U.S. Department of Labor Employment and Training Administration. The product was created by the grantee and does not necessarily reflect the official position of the U.S. Department of Labor makes no guarantees, warranties, or assurances of any kind, express or implied, with respect to such information, including any information on linked sites and including, but not limited to, accuracy of the information or its completeness, usefulness, usefulness, adequacy, continued availability, or ownership.



Unless otherwise noted this MoSTEMWINs material by St. Louis Community College is licensed under a Creative Commons Attribution 4.0 International License.

St. Louis Community College	Adult Learning Academy Elementary Algebra Workbook 5.1 SIMPLIFYING EXPRESSIONS	
	Simplify each expression:	
1. $x^6 \cdot x^2$	11. 5 <sup>-2</sup>	22. $\left(\frac{5}{x}\right)^{-2}$
2. $x^6 + x^2$	12. 0 <sup>5</sup>	4.9
3. $x^6 \div x^2$	13. 0 <sup>0</sup>	23. $\frac{12x^4x^8}{4x^3}$
4. $(x^6)^2$	14. $-5^{-2}$	24. $\frac{-10x^5y^{-3}}{15x^{-3}y^2}$
5. $10x^8 \cdot 2x^2$	15. $(-5)^2$	
6. $(10x^8)^2$	16. (-5) <sup>-2</sup>	25. $\frac{4x^7x^{-3}y^7}{4x^5y^6}$
7. $\frac{10x^8}{2x^2}$	17. $5x^{-2}$	26. $\left(\frac{7x^5y^{-2}}{14x^{-3}y^4}\right)^3$
8. $10x^8 - 2x^2$	18. $(5x)^{-2}$	
9. $10x^8 - 2x^8$	19. x <sup>0</sup>	27. $\left(\frac{12x^{-2}y^4}{4x^{-3}y^{-3}}\right)^{-3}$
105 <sup>2</sup>	20. $5x^0$	
	21. $(5x)^0$	

MOSTEM



## Adult Learning Academy Elementary Algebra Workbook 5.2 COLOR MATCHING SIMPLIFIED EXPRESSIONS



Simplify each expression, and color the matching simplified expressions.





#### Adult Learning Academy Elementary Algebra Workbook 5.3 MULTIPLYING POLYNOMIALS



1. $x \cdot x$	11. $(x + 7)(x - 5)$	21. $(3x-5)^2$
2. $\mathbf{x} \cdot \mathbf{x}^2$	12. $(x + 7)(x - 5)$	22. $(2x + 7)^2$
3. $x^2 \cdot x^3$	13. $(x + 7)(x - 7)$	23. $(3x-5)(3x+5)$
4. $x^5 \cdot x^2$	14. $(x-5)(x+5)$	24. $(2x + 7)(2x - 7)$
5. $5x^3 \cdot -2x^4$	15. $(x + 7)^2$	25. $(x + y)(x - y)$
6. $-6x^5 \cdot -4x^3$	16. (x - 7) <sup>2</sup>	26. $(x + y)^2$
7. $3x(4x^2 - 5x + 1)$	17. $(x + 5)^2$	27. $(x - y)^2$
8. $-2x^3(5x^4-3)$	18. $(x - 5)^2$	28. $(3x + 2y)(3x - 2y)$
9. $7x^2(x^3 - 3x + 2)$	19. $(3x + 5)(2x - 7)$	29. $(3x + 2y)^2$
10. $(x + 7)(x + 5)$	20. $(3x-5)(2x+7)$	30. $(3x - 2y)^2$



## Adult Learning Academy Elementary Algebra Workbook 5.4 SCIENTIFIC NOTATION



Fill in the table:

Item	Scientific Notation	Standard Notation
Approximate number of hairs on your head		140,000
Number of cells in your brain		100,000,000,000
Length of a rhinovirus in meters		.00000020
Number of heartbeats in a lifetime	2.7 x 10 <sup>9</sup>	
Speed that human hair grows in miles per hour	1.0 x 10 <sup>-8</sup>	
Number of red blood cells in a human body	$3.0 \ge 10^{13}$	
Lung capacity of a blue whale in milliliters		5,000,000
Speed of a snail in kilometers per second		.000013
Thickness of a sheet of paper in inches	2.75 x 10 <sup>-3</sup>	





Expression	CHOOSE: Monomial, Binomial, Trinomial, Polynomial	Degree
$3x^2 - 2x + 1$		
5xyz		
4x + 2y		
$5x^2y - 11$		
139		
$-642x^{39}$		
2y + 3x - 5w + p		
$57x^5 - 2x^3 + 11x$		
X		
$4x^2 + 3x + x$		
$2x + y + z^{12}$		
5x <sup>0</sup>		
$100x^2 - py^3$		
3x + 2		
10x - y + z + p - 5		

- 1. Create a 2<sup>nd</sup>-degree trinomial:
- 2. Create a 4<sup>th</sup>-degree monomial:



3. Create a 3<sup>rd</sup>-degree binomial:

- 4. Can the sum of two binomials ever be a trinomial? If so, show an example:
- 5. Can the sum of two binomials ever be a monomial? If so, show an example:
- 6. Can the sum of two binomials ever be a binomial? If so, show an example:
- 7. Can the product of two binomials ever be a binomial? If so, show an example:
- 8. Can the product of two binomials ever be a trinomial? If so, show an example:
- 9. Can the product of two binomials ever have four terms? If so, show an example:



# Adult Learning Academy Elementary Algebra Workbook 5.6 Evaluating, Adding, and Subtracting Polynomials



Here are three functions:

$$f(x) = 3x^2 - 2x + 1$$
  $g(x) = -2x^2 - 5$   $h(x) = -4x + 2$ 

To "evaluate" means to plug in the value of x and see what you get. When you plug in a negative number for x, always put parentheses around it!

- 1. Evaluate the following:
  - a) f(2) b) g(-3) c) h(0)
  - d) f(-5) e) g(0) f) h(-5)
- 2. When you add and subtract polynomials, combine like terms. When you subtract, be sure to subtract EVERY term!
  - a) f(x) + g(x) b) g(x) + h(x) c) f(x) + h(x)

d) f(x) - g(x) e) g(x) - h(x) d) f(x) - h(x)

- 3. You can also multiply every term of a polynomial by a number. Find these:
  - a) 4f(x) + 3g(x) b) 5f(x) 2h(x)



# Adult Learning Academy Elementary Algebra Workbook 5.7 CAREER APPLICATIONS: STEM



1. The day's revenue (income) for a computer company depends on how many clients come to get their computers fixed. The revenue can be modeled by the function

 $R(x) = x^3 - x^2$  where R(x) is the revenue in dollars, and x is the number of clients that day

The day's costs (expenses) for the same company also depend on how many clients come to get their computers fixed. The costs can be modeled by the function

 $C(x) = .75x^3 - .7x^2 - .5x + 10$  where C(x) is the cost in dollars, and x is the number of clients that day

Any business calculates its PROFIT by starting with Revenue (income) and subtracting expenses (costs). So the profit function, P(x), can be modeled by

$$\mathbf{P}(\mathbf{x}) = \mathbf{R}(\mathbf{x}) - \mathbf{C}(\mathbf{x}).$$

a. For this company, what is the profit function? (subtract R(x) - C(x)):

- b. If 3 clients come to get their computers fixed on a given day, what is the Revenue? The Cost? The Profit?
- c. If 20 clients come to get their computers fixed on a given day, what is the Revenue? The Cost? The Profit?
- d. Say that Revenue doubles (get multiplied by 2), but costs remain the same. Show the new function for Profit:

2. Write an algebraic expression for the PERIMETER (add all the sides) and the AREA (length times width) of each rectangle on the page. Remember that area is measured in square units.



3. Barnes-Jewish Hospital in St. Louis is 177 feet tall. If you stood at the top of the hospital and dropped a penny (NOTE: this is NOT recommended!), the following function tells you high off the ground the penny would be after t seconds:

$$h(t) = -16t^2 + 177$$

where t is the number of seconds since you dropped the penny, and h(t) is the penny's height in feet off the ground.



- a. Find h(0). What does this information tell you?
- b. Find h(1), the height of the penny 1 second after being dropped:
- c. Find h(2), the height of the penny 2 seconds after being dropped:
- d. Find h(3), the height of the penny 3 seconds after being dropped:
- e. Would the penny still be falling 4 seconds after being dropped? How do you know?



# Adult Learning Academy Elementary Algebra Workbook MODULE 5 ANSWER KEY



5.1 Simplifying Expressions	5.2 Color Matching Expressions
1. x <sup>8</sup>	$2^0 = \frac{3x}{2} = 1$
2. not like terms	3x 2x = (52-4+2) = (52-4+4) = 2-0
3. x <sup>4</sup>	$\frac{-}{x} = (53x^2 + 3) - (53x^2 + 1) = 2x^2$
4. $x^{12}$	$-5^2 = 0x - 25 = -25$
5. $20x^{10}$	$\mathbf{x} + \mathbf{x} = 5\mathbf{x} - 3\mathbf{x} = 2\mathbf{x}$
6. 100x <sup>16</sup>	
7. $5x^6$	$100x^2 - 99x^2 = x \cdot x = x^2$
8. not like terms	$3x^2 - x^2 = \frac{10x^5}{5x^3} = 2x^2$
9. $8x^8$	$\frac{x^{3}+x}{x^{2}+x} = x^{2} + 1 = (5x^{2} - 1) - (4x^{2} - 2) = x^{2} + 1$
1025	
11. 1/25	$(x+1)(x-1) = (5x^2-2) - (4x^2-1) = x^2 - 1$
	$(x+1)^2 = x^2 + 2x + 1$
13. undefined	$(\mathbf{x} - 1)^2 = \frac{5x^2 - 10x + 5}{2x^2 - 10x + 5} = \mathbf{x}^2 - 2\mathbf{x} + 1$
14. $\frac{1}{25}$	$(x-1) - \frac{5}{5} - x - 2x + 1$
15. 25	5.3 Multiplying Polynomials
16. $\frac{1}{27}$	1. x <sup>2</sup>
5	2. $x^3$
$17.\frac{3}{x^2}$	3. x <sup>5</sup>
$18 \frac{1}{1}$	4. x'
$25x^2$	510x'
19.1	6. 24x°
20.5	7. $12x^3 - 15x^2 + 3x$
21.1	<b>8.</b> $-10x^2 + 6x^2$ <b>9.</b> $7x^5 - 21x^3 + 14x^2$
22. $\frac{x^2}{25}$	9. $7x^2 - 21x^2 + 14x^2$ 10. $x^2 + 12x + 25$
25 23 $3v^9$	10. x + 12x + 55 $11 x^2 + 2x = 35$
23. 34	11. $x + 2x - 35$ 12. $x^2 - 12x + 35$
$24. \frac{-2x}{3y^5}$	$12. x^{2} - 49$
25 <sup>y</sup>	14. $x^2 - 25$
$\frac{25 x}{x}$	<b>15.</b> $x^2 + 14x + 49$
$26.\frac{x^{24}}{x^{24}}$	<b>16.</b> $x^2 - 14x + 49$
8y <sup>18</sup>	<b>17.</b> $x^2 + 10x + 25$
$27.\frac{1}{27x^3x^{21}}$	<b>18.</b> $x^2 - 10x + 25$
$2ix^{-}y^{}$	<b>19.</b> $6x^2 - 11x - 35$
$\frac{20.5 \text{ spin up the fraction}}{x^4 + x^3 + 2x + 5}$	<b>20.</b> $6x^2 + 11x - 35$
$\frac{1}{3}$ - $\frac{1}{5}$ + $\frac{1}{3}$ - $\frac{1}{3x}$	$21.\ 9x^2 - 30x + 25$

Module 5: Integer Exponents and Laws, Scientific Notation, Polynomials and Operations

# 5.3 Multiplying Polynomials (cont.)

22.  $4x^2 + 28x + 49$ 23.  $9x^2 - 25$ 24.  $4x^2 - 49$ 25.  $x^2 - y^2$ 26.  $x^2 + 2xy + y^2$ 27.  $x^2 - 2xy + y^2$ 28.  $9x^2 - 4y^2$ 29.  $9x^2 + 12xy + 4y^2$ 30.  $9x^2 - 12xy + 4y^2$ 

# 5.4 Scientific Notation

Item	Scientific Notation	Standard Notation
Number of hairs on your head	1.4 x 10 <sup>5</sup>	140,000
Number of cells in your brain	1.0 x 10 <sup>11</sup>	100,000,000,000
Length of a rhinovirus in meters	2.0 x 10 <sup>-8</sup>	.00000020
Number of heartbeats in a lifetime	2.7 x 10 <sup>9</sup>	2,700,000,000
Speed that human hair grows in miles per hour	1.0 x 10 <sup>-8</sup>	.00000001
Number of red blood cells in a human body	3.0 x 10 <sup>13</sup>	30,000,000,000,000
Lung capacity of a blue whale in mL	5.0 x 10 <sup>6</sup>	5,000,000
Speed of a snail in kilometers per second	1.3 x 10 <sup>-5</sup>	.000013
Thickness of a sheet of paper in inches	2.75 x 10 <sup>-3</sup>	.00275

## 5.5 Thinking About Polynomials

Expression	CHOOSE:	Degree
$3x^2 - 2x + 1$	Trinomial	2 <sup>nd</sup>
5xyz	Monomial	3rd
4x + 2y	Binomial	1 <sup>st</sup>
5x <sup>2</sup> y - 11	Binomial	3rd
139	Monomial	0 degree
-642x <sup>39</sup>	Monomial	39 <sup>th</sup>
2y + 3x - 5w + p	Polynomial	1 <sup>st</sup>
$57x^5 - 2x^3 + 11x$	Trinomial	5 <sup>th</sup>
х	Monomial	1 <sup>st</sup>
$4x^2 + 3x + x$	Trinomial	2 <sup>nd</sup>
$2x + y + z^{12}$	Trinomial	12 <sup>th</sup>
5x <sup>0</sup>	Monomial	0 degree
$100x^2 - py^3$	Binomial	4 <sup>th</sup>
3x + 2	Binomial	1 <sup>st</sup>
10x - y + z + p - 5	Polynomial	1 <sup>st</sup>

# 5.5 Thinking About Polynomials (cont.)

answers will vary, ex. x<sup>2</sup> + 3x - 5
 answers will vary, ex. 5x<sup>4</sup>
 answers will vary, ex. 5x<sup>3</sup> - 3x
 Yes, (x + 3) + (y + 5) = x + y + 8
 Yes, (x + 3) + (x - 3) = 2x
 Yes, (x + 3) + (x + 5) = 2x + 8
 Yes, (x + 3)(x - 3) = x<sup>2</sup> - 9
 Yes, (x + 3)(x + 2) = x<sup>2</sup> + 5x + 6
 Yes, (x + 3)(y + w) = xy + xw + 3y + 3w

## 5.6 Evaluating, Adding, and Subtracting Poly.

- 1a. f(2) = 3(4) 2(2) + 1 = 91b. g(-3) = -2(9) - 5 = -231c. h(0) = -4(0) + 2 = 21d. f(-5) = 3(25) - 2(-5) + 1 = 861e. g(0) = -2(0) - 5 = -51f. h(-5) = -4(-5) + 2 = 22
- 2a.  $x^2 2x 4$ 2b.  $-2x^2 - 4x - 3$ 2c.  $3x^2 - 6x + 3$ 2d.  $5x^2 - 2x + 6$ 2e.  $-2x^2 + 4x - 7$ 2f.  $3x^2 + 2x - 1$
- **3a.**  $12x^2 8x + 4 6x^2 15$ =  $6x^2 - 8x - 11$ **3b.**  $15x^2 - 10x + 5 + 8x - 4$ =  $15x^2 - 2x + 1$

## 5.7 Career Applications: STEM

- 1a.  $R(x) C(x) = x^3 x^2 (.75x^3 .7x^2 .5x + 10)$ =  $x^3 - x^2 - .75x^3 + .7x^2 + .5x - 10$ =  $.25x^3 - .3x^2 + .5x - 10$ 1b.  $R(3) = 3^3 - 3^2 = 27 - 9 = $18$
- **10.**  $\mathbf{R}(3) = 3^3 3^2 = 27 9 = \$18$   $\mathbf{C}(3) = .75(3^3) - .7(3^2) - .5(3) + 10$  = .75(27) - .7(9) - 1.5 + 10 = 20.25 - 6.3 - 1.5 + 10 = \$22.45
  - P(3) = 18 22.45 = -\$4.45 (loss)
- **1c.**  $\mathbf{R}(\mathbf{20}) = 20^3 20^2 = 8000 400 = \$7600$   $\mathbf{C}(\mathbf{20}) = .75(20^3) - .7(20^2) - .5(20) + 10$  = .75(8000) - .7(400) - 10 + 10 = 6000 - 280 = \$5720 $\mathbf{P}(\mathbf{20}) = 7600 - 5720 = \$1880$

Module 5: Integer Exponents and Laws, Scientific Notation, Polynomials and Operations

#### 5.7 Career Applications: STEM (cont.)

1d. P(x) = 2R(x) - C(x) $= 2x^3 - 2x^2 - (.75x^3 - .7x^2 - .5x + 10)$  $= 1.25x^3 - 1.3x^2 + .5x - 10$ **2a.** P = x + x + x + x = 4x in.  $A = x * x = x^2$  sq. in. **2b.**  $\mathbf{P} = \mathbf{x} + (2\mathbf{x} + 1) + \mathbf{x} + (2\mathbf{x} + 1)$ = 6x + 2 ft.A = x(2x + 1) $= 2x^2 + x \text{ sq. ft.}$ **2c.**  $\mathbf{P} = 3x + (x - 5) + 3x + x - 5$ = 8x - 10 cm A = 3x(x - 5) $= 3x^2 - 15x$  sq. cm **2d.**  $\mathbf{P} = (x+5) + (x-3) + (x+5) + (x-3)$ **= 4x + 4 meters** A = (x + 5)(x - 3) $= x^2 - 3x + 5x - 15$  $= x^{2} + 2x - 15$  sq. meters **2e.**  $\mathbf{P} = (x + 7) + (x - 7) + (x + 7) + (x - 7)$ = 4x miles A = (x + 7)(x - 7) $= x^{2} - 49$  sq. miles **2f.**  $\mathbf{P} = (x + 5) + (x + 5) + (x + 5) + (x + 5)$ = 4x + 20 kmA = (x + 5)(x + 5) $= x^{2} + 10x + 25$  sq. km

3a. h(0) = -16(0)<sup>2</sup> + 177 = 177 This tells us that the penny is 177 feet off the ground (on top of the building) when you haven't thrown it yet
3b. h(1) = -16(1)<sup>2</sup> + 177 = -16 + 177 = 161 feet.

**3c.**  $h(2) = -16(2)^2 + 177 = -64 + 177 = 113$  feet

**3d.**  $h(3) = -16(3)^2 + 177 = 33$  feet

**3e.** No; h(4) = -16(4)<sup>2</sup> + 177 = -79 feet (underground?)!



This product is 100% funded by the MoSTEMWINS \$19.7 million grant from the U.S. Department of Labor Employment and Training Administration. The product was created by the grantee and does not necessarily reflect the official position of the U.S. Department of Labor. The Department of Labor makes no guarantees, warranties, or assurances of any kind, express or implied, with respect to such information, including any information on linked sites and including, but not limited to, accuracy of the information or its completeness, timeliness, usefulness, adequacy, continued availability, or ownership.



Unless otherwise noted this MoSTEMWINs material by <u>St. Louis Community College</u> is licensed under a <u>Creative</u> <u>Commons Attribution 4.0 International License</u>.