Adult Learning Academy Elementary Algebra Workbook

Module 4: Equations of lines, graphs of Linear inequalities, SOLVING SYSTEMS BY GRAPH

## Learning ObJectives

By the time you finish this module, you should be able to:Given a point and a slope, graph and write the equation of the line.
$\square$ Given two points, graph and write the equation of a line containing them.
$\square$ Write the equation of a line in standard form, in point-slope form, and in slopeintercept form.
$\square$ Graph a system of two equations and find the solution by locating their intersection.
$\square$ Recognize and graph parallel lines.
$\square$ Recognize and graph perpendicular lines.
$\square$ Find the slope of parallel and perpendicular lines.
$\square$ Graph linear inequalities by shading.

## IMPORTANT INFORMATION FROM MODULE 4:

SLOPE: Given any two points $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$, the slope of the line containing them is $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$.

SLOPE-INTERCEPT FORM OF A LINEAR EQUATION:
$y=m x+b$, where $m$ is the slope and $b$ is the y -intercept.
POINT-SLOPE FORM OF A LINEAR EQUATION:
$y-y_{1}=m\left(x-x_{1}\right)$, where $m$ is the slope and $\left(x_{1}, y_{1}\right)$ is a point on the line.
STANDARD FORM OF A LINEAR EQUATION:
$A x+B y=C$, where $A, B$, and $C$ are constants.
PARALLEL LINES have equal slopes.
PERPENDICULAR LINES have opposite, reciprocal slopes.
TO GRAPH A LINEAR INEQUALITY: 1) draw the border line solid for > or <, dotted for > or <;
2) test a point not on the line; 3) shade the "good" side of the line


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## Adult Learning Academy <br> Elementary Algebra Workbook <br> 4.1 How to Write the equation of a Line

| Standard Form: | Slope-Intercept Form: | Point-Slope Form: | Slope Formula: |
| :---: | :---: | :---: | :---: |
| $\mathbf{A x}+\mathbf{B y}=\mathbf{C}$ | $\mathbf{y}=\mathbf{m x}+\mathbf{b}$ | $\mathbf{y}-\mathbf{y} 1=\mathbf{m}(\mathbf{x}-\mathbf{x} 1)$ | $\mathbf{m}=\frac{y_{2-y_{1}}}{x_{2-x_{1}}}$ |


| If you know the SLOPE and the Y-INTERCEPT, use the SLOPE-INTERCEPT FORM | If you know the SLOPE and any old POINT, use the POINT-SLOPE FORM | If you know 2 POINTS that are on the line, use them to find the SLOPE. Then pick one of the points and use the POINT-SLOPE FORM |
| :---: | :---: | :---: |
| 1a. Write the equation of a line with a slope of $-1 / 22$ and $y$-intercept at ( 0,3 ). | 2a. Write the equation of a line with a slope of $-1 / 2$ that passes through the point $(4,7)$ | 3a. Write the equation of a line that contains the points $(3,7)$ and $(-2,1)$. |
| 1b. Write the equation of a line with a slope of $3 / 4$ and $y$-intercept at $(0,-2)$. | 2 b . Write the equation of a line with a slope of $3 / 4$ that passes through the point $(-2,5)$ | 3b. Write the equation of a line that contains the points $(0,5)$ and $(2,-3)$. |

## Adult Learning Academy <br> Elementary Algebra Workbook <br> 4.2 Parallel and Perpendicular lines

1. Using a ruler, carefully graph each of these lines on the one coordinate system below. What do you notice?
a. $y=1 / 2 x+3$
b. $y=1 / 2 x-2$
c. $y=1 / 2 x$
d. $y=1 / 2 x+5$
e. $y=1 / 2 x-4$
f. $y=1 / 2 x-8$

2. PARALLEL lines never intersect (cross). Parallel lines have equal slopes. Determine if each pair of lines below is parallel.
a. $y=-3 x+1$ and $y=-3 x-4$
b. $y=3 x+5$ and $y=2 x+5$
c. $y=3 x+5$ and
d. $x+y=5$ and $x+y=10$
e. $2 x+3 y=6$ and $y=-2 / 3 x+4$
f. $x=5$
and $\quad y=5$

Are they parallel? $\qquad$

Are they parallel? $\qquad$

Are they parallel? $\qquad$

Are they parallel? $\qquad$

Are they parallel? $\qquad$

Are they parallel? $\qquad$

### 4.2 PARALLEL AND PERPENDICULAR LINES (CONT.)

3. Using a ruler, carefully graph each of these lines on the one coordinate system below. What do you notice?

$$
y=\frac{3}{4} x+3 \quad y=\frac{-4}{3} x+1
$$


4. PEPENDICULAR lines intersect (cross) at a right angle ( 90 degrees). Perpendicular lines have OPPOSITE (one positive, one negative), RECIPROCAL (upside-down fractions) slopes.

For example, if one line has a slope of $\frac{1}{3}$, a line perpendicular to it would have slope $\frac{-3}{1}$ or -3 .

For each pair of equations below, circle whether the two lines are parallel (same slope), perpendicular (opposite reciprocal slopes), or neither.
a. $y=-3 x+1$ and $y=-3 x-4 \quad$ Parallel Perpendicular Neither
b. $y=3 x+5$ and $y=2 x+5 \quad$ Parallel Perpendicular Neither
c. $y=\frac{2}{3} x+5$ and $y=\frac{-3}{2} x+2 \quad$ Parallel $\quad$ Perpendicular Neither
d. $y=4 x+1$ and $y=-4 x+1 \quad$ Parallel Perpendicular Neither
e. $y=4 x+1 \quad$ and $y=\frac{-1}{4} x \quad$ Parallel $\quad$ Perpendicular Neither
f. $3 x+2 y=6$ and $\quad y=\frac{2}{3} x+4 \quad$ Parallel Perpendicular Neither
g. $x=5$ and $\quad y=5 \quad$ Parallel Perpendicular Neither

In each problem, you are given the equation of a line. Create the equation for any line parallel to the first line, and the equation for any line perpendicular to the first line. Finally, graph all three lines on the same coordinate system.

5. Original line: $y=1 / 4 x-2$
a. Parallel line: $\qquad$
b. Perpendicular line: $\qquad$
c. Graph all three lines to the left.

6. Original line: $y=1 / 4 x-2$
a. Parallel line: $\qquad$
b. Perpendicular line: $\qquad$
c. Graph all three lines to the left.

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A "system" of equations is just more than one equation that must be true at the same time.

For example, here is a system of two equations: $\quad \mathbf{x}+\mathbf{y}=7$

$$
2 x+y=10
$$

There are infinitely many solutions to the equation $x+y=7$. For example, the points $(0,7),(1,6),(2,5),(3,4),(4,3),(5,2),(6,1),(7,0),(8,-1)$, and $(9,-2)$ all satisfy the equation and are on the line $\mathrm{x}+\mathrm{y}=7$.

Similarly, there are infinitely many solutions to the equation $2 \mathrm{x}+\mathrm{y}=10$. For example, the points ( 0 , $10),(1,8),(2,6),(3,4),(4,2),(5,0),(6,-2),(7,-4)$, and $(8,-6)$ all satisfy the equation and are on the line $2 \mathrm{x}+\mathrm{y}=10$.

To solve the SYSTEM of equations means to find the point that satisfies BOTH equations in the system. In other words, we are looking for a point that is on BOTH lines. It's where the lines INTERSECT (cross)!

1. Graph the lines $\mathbf{x}+\mathbf{y}=\mathbf{7}$ and $\mathbf{2 x + y}=\mathbf{1 0}$ below and see if you can find the point where they cross. That's the solution to the system. In future courses you will even learn how to find this point without graphing!


### 4.3 Systems of Equations (cont.)

Use a ruler to carefully graph each pair of lines on the one coordinate system and find their intersection. The point of intersection is the solution to the system of equations.
2.

3.

4.


$$
y=1 / 2 x-1
$$

$$
y=-x-7
$$

Check by plugging the $x$ and $y$ coordinates of your solution into each equation. They should both be satisfied! Did you find the $x$ and $y$ values that make BOTH equations true?
$y=x-7$
$y=-x+3$

Does your solution check?
$y=\frac{2}{3} x-4$
$y=\frac{2}{3} x+1$

What do you notice about these lines? Where do they cross?

## Adult Learning Academy Elementary Algebra Workbook 4.4 GRAPHING LINEAR INEQUALITIES

a. $\mathrm{X}+\mathrm{Y}=7$



d. $\mathrm{X}+\mathrm{Y} \leq 7$
e. $\quad \mathrm{X}+\mathrm{Y} \geq 7$
f. $\quad Y>2 x-5$




## RECIPE for graphing linear inequalities:

1. Temporarily pretend that there is an equal sign.
2. Graph the border line: use a solid line for $\geq$ or $\leq$, use a dotted line for $>$ or $<$.
3. Choose a test point NOT on the border. Most people choose $(0,0)$ as long as it's not on the border.
4. Plug the $x$-coordinate and the $y$-coordinate of the test point into the original inequality. If the point satisfies the inequality, then shade the side containing that point. If the point does NOT satisfy the inequality, then shade the other side.

### 4.4 GRAPHING LINEAR INEQUALITIES (CONT.)

g. $y<1 / 4 x-2$

j. $\quad y \geq 7$

m. $y>-3 / 4 x+2$

h. $y>3 x$

k. $y<-2 / 3 x+1$

n. $y-x \leq 3$

i. $\mathrm{y} \leq 1 / 2 \mathrm{x}+3$

l. $\mathrm{x}<4$

o. $3 x+5 y<15$


## POINT-SLOPE FORM

(to the tune of Ghostbusters), lyrics by Debbie Char

> When you've got a point
> And you know the slope,
> Whatcha gonna use?
> POINT-SLOPE FORM!!!

> When you've got TWO points, You can find the slope, then Whatcha gonna use? POINT-SLOPE FORM!!!

Y minus $\mathrm{Y}_{1}$, which is the y-coordinate, Equals m, which is the slope, then in parentheses X minus $\mathrm{X}_{1}$, which is the x -coordinate, That's the Point-Slope form of the equation of a line!

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4.6 Point-Slope Form Practice

$$
\mathbf{y}-\mathbf{y}_{1}=\mathbf{m}\left(\mathbf{x}-\mathbf{x}_{1}\right)
$$

|  | What You Know | Equation in Point-Slope Form | Sketch Graph |  |
| :---: | :---: | :---: | :---: | :---: |
| A. | Slope $m=1 / 2$ <br> Point (3, 1) |  |  |  |
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|  |  |  |  |  |
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|  |  |  |  |  |
| B. | Slope $m=1 / 2$ <br> Point ( 3,1 ) |  |  |  |
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|  |  |  |  |  |
|  |  |  |  |  |
|  | Slope $m=1 / 2$ <br> Point (3, 1) |  |  |  |
| C. |  |  |  |  |
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|  |  |  |  |  |
| D. | Slope $m=1 / 2$ <br> Point (3, 1) |  |  |  |
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| E. | Slope $m=1 / 2$ <br> Point ( 3,1 ) |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  | \# | \# |
|  |  |  | \# |  |
|  |  |  |  |  |
|  |  |  | $\ldots$ |  |

4.6 Point-Slope Form Practice (CONT.)


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1. A farmer starts with 40 acres, and plants two additional acres each year.
a. Graph the number of acres planted over the years. Why is it only shown in the first quadrant?

b. What is the slope of this line? Why is it positive? What is the $y$-intercept?
c. Write the equation of the line. What form of the linear equation did you choose?
d. Another farmer starts at 44 acres, and also plants 2 additional acres each year. Make this line in another color on the same graph.
e. Write the equation for the second farmer's acres planted. What do you notice about the two lines? How can you tell that this is true from their equations?
f. A third farmer starts with 40 acres planted, but each year plants 3 more. Graph her line in a third color.
g. Compare this new line with the other two. What is the same? What is different? Explain what you see.
h. If all three farmers continue in their patterns for 30 years, how much will each be planting in their $30^{\text {th }}$ year?
2. A generous long-term grant gives a research team $\$ 100$ million. Each year they spend 5 million dollars of the grant.
a. Graph the amount left on the grant over the first six years:

b. What is the slope of this line? Is it positive or negative? Why?
c. Write the equation for the line. What form did you choose?
d. Another lab received a grant for $\$ 80$ million, and will also use $\$ 5$ million per year. Write the equation for the amount of money left in the grant each year, and graph it in another color above. What can you say about the two lines?
e. A third lab received a grant for $\$ 100$ million, but will use $\$ 10$ per year. Write the equation for its value over time, and graph it in another color above. Compare this equation to the others. What is the same? What is different?
f. After 10 years, how much of each grant will remain?
3. A researcher who has been with the lab 5 years makes $\$ 55,000$ per year. A researcher who has been with the lab 7 years makes $\$ 65,000$ per year.
a. Use the two points described above to graph a line:

b. What is the slope of this line?
c. Write the equation of the line. What form did you use?
d. According to your equation, how much would a researcher make annually after working 25 years?
e. According to your equation, how much would a starting researcher make?
4. A lab can hire a maximum of 20 technicians. Some can be part-time and others can be full-time. If $x=$ the number of part-timers and $y=$ the number of full-timers, the inequality $x+y \leq 20$ describes the situation.
a. Graph all possible combinations of part-time and full-time drivers:
b. Can the lab hire 10 part-time and 10 full-time technicians? Where is this point?

c. Can the company hire 5 part-time and 5 full-time technicians? Where is this point?
d. Can the company hire 18 part-time and 5 full-time technicians? Where is this point?
5. The Human Resources Department at work offers you a choice of two prescription drug plans. Plan A charges $\$ 50$ per year plus $\$ 10$ per prescription filled. Prescription Plan B charges $\$ 25$ per year plus $\$ 15$ per prescription filled.
a. Graph and label each equation.

$$
\begin{aligned}
& \mathrm{A}(\mathrm{x})=50+10 \mathrm{x} \\
& \mathrm{~B}(\mathrm{x})=25+15 \mathrm{x}
\end{aligned}
$$


b. Where do the two lines cross? Interpret the meaning of that point.

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Module 4 Answer Key

### 4.1 How to Write the Equation of a Line

1a. $y=-1 / 2 x+3$
1b. $y=3 / 4 x-2$

2a. $y-7=-1 / 2(x-4)$
2b. $y-5=3 / 4(x+2)$

3a. $m=\frac{1-7}{-2-3}=\frac{-6}{-5}=6 / 5$

$$
y-7=6 / 5(x-3) \quad \text { OR } \quad y-1=6 / 5(x+2)
$$

3a. $\mathrm{m}=\frac{-3-5}{2-0}=\frac{-8}{2}=-4$
$y-5=-4 x \quad$ OR $\quad y+3=-4(x-2)$

### 4.2 Parallel and Perpendicular Lines

1. 


4a. Parallel
4b. Neither
4c. Perpendicular
4d. Neither
4e. Perpendicular
4f. Perpendicular
4g. Perpendicular

2a. Yes
2b. No
2c. No
2d. Yes
2e. Yes
2f. No
3.


5a. may vary: $y=1 / 4 x+3$ (slope must equal $1 / 4$ )
5b. may vary: $\mathrm{y}=-4 \mathrm{x}+1$ (slope must equal -4)

5c.

3. $(5,-2)$

4. Parallel lines; do not cross


### 4.4 Graphing Linear Inequalities

a.

e.

i.

m.

n.


### 4.6 Point-Slope Practice

a. $y-1=1 / 2(x-3)$

e. $y-1=3 / 4(x-0) O R$ $y=3 / 4 x+1$

b. $y-4=-2 / 3(x+5)$

f. $\quad m=3 / 4$
$y-5=3 / 4(x-3)$ OR $y-2=3 / 4(x+1)$

0.

c.

g.

k.

c. $y+4=2(x+3)$

g. $m=-1$
$y-3=-1(x+4) O R$ $y+2=-1(x-1) O R$ $y=-x-1$

d. $y-1=0(x-3) O R$ $y=1$

h. $y+3=2 x-2$
$y=2 x-5$


### 4.6 Point-Slope Practice (cont.)

i. $y+3=2 x-2$
$y=2 x-5$

j. $y-2=1 / 2 x+3$
$y=1 / 2 x+5$


### 4.7 Career Applications: STEM

1a. Acres and time only make sense in positive quantities


1b. $m=2, y$-int. $=40$; slope is positive because the more years go by, the more acres are planted
1c. slope-intercept form $y=2 x+40$
1d. see graph above
1e. $y=2 x+44$; lines a and $d$ are parallel because they have the same slope
1f. $y=3 x+40$
1g. has same $y$-intercept as line $a$, but has steeper incline
1h. $y=2(30)+40=100$ acres
$y=2(30)+44=104$ acres
$y=3(30)+40=130$ acres

2a.


2b. $m=-5 / 1$; as years increase, money decreases
2c. slope-intercept form $y=-5 x+100$
k. $y+1=3 x+6$
$y=3 x+5$


2d. $y=-5 x+80$; lines a and $c$ are parallel because they have the same slope
2e. $y=-10 x+100$; same $y$-intercept as line a but steeper decline
2f. $y=-5(10)+100=\$ 50$ million left
$y=-5(10)+80=\$ 30$ million left
$y=-10(10)+100=\$ 0$ left
3a.


3b. $m=\frac{65,000-55,000}{7-5}=\frac{10,000}{2}=5,000$
3c. point slope form $y-65,000=5,000(x-7) O R$ point slope form $y-55,000=5,000(x-5) O R$ slope-intercept form $\mathrm{y}=5000 \mathrm{x}+\mathbf{3 0 , 0 0 0}$
3d. $\mathrm{y}=5000(25)+\mathbf{3 0 , 0 0 0}=\$ 155,000$
3e. $y=5000(0)+30,000=\$ 30,000$
4a. $\mathrm{y} \leq-\mathrm{x}+20$


4b. Yes, the point is on the line
4c. Yes, this point is in the shaded area
4d. No, this point is not in the shaded area

### 4.7 Career Applications: STEM (cont.)

5 a.


5 . They intersect at $(5,100)$; if you get 5 prescriptions, the cost will be the same with either plan

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