Ch. 9 Part Name________________________

Handout - Half-Life &Exponential Growth

Here are two ways to predict growth, whether it be population growth, the spread of a disease, etc.

1) Doubling Time Growth Model

One measure of growth rate deals with the time it takes for a population to double. Over short periods the **doubling time growth model** is as follows:

$$\boldsymbol{A = A_0 2^{\frac{t}{d}}}$$

A = Population at time t

A_0 is Population at time 0 (a specific time, when a known population exists)

d = Doubling time

Example: Venezuela has a population of about 30 million and it is estimated that the population will double every 20 years. What will be the population in 12 years? What will be the population in 40 years?

$A = 30(2)^{\frac{12}{20}} \approx 45.47\, million$

$A = 30(2)^{\frac{40}{20}} \approx 120\, million$

2) Relative Growth Rate

The **relative growth rate model** is an exponential model that also predicts growth. Notice that this is basically the same formula used to find interest that is compounded continuously.

$$\boldsymbol{A = A_0 e^{kt}}$$

A = Population at time t

A_0 is Population at time 0 (a specific time, when a known population exists)

k=relative growth rate

e is an irrational number (kind of like pi) that is found on all scientific calculators

Example: A city has a population of 350,000 and its population is growing continuously at a relative growth rate of 1.56%. What will be the city's population in 15 years?

$A = 350{,}000(e)^{(.0156*15)} \approx 442{,}276\, people$

Half-life (Negative Exponential Growth)

Radioactive materials are used extensively in medical diagnosis and therapy, and also as power sources in satellites. The rate of decay is often measured by the half-life of the material. Half-Life is the time it takes for half of the particular material to decay. The model is:

$$A = A_0\left(\frac{1}{2}\right)^{t/h}$$

$A = Amount\ \ at\ \ time\ \ t$

$A_0 = Amount\ \ at\ \ time\ \ t = 0$

$h = half\ -life$

Example: The radioactive isotope gallium 67, used in the diagnosis of malignant tumors, has a biological half-life of 46.5 hours. If we start with 100 milligrams of the isotope, how many milligrams will be left after a week? (Notice we will use 168 hours for time, since the half-life is given in hours. So we must convert one week to 168 hours).

$A = 100(.5)^{\left(\frac{168}{46.5}\right)} \approx 8.17\ milligrams$