Binary Numbering System

Numbering systems are fascinating. The traditional numbering system we use is **Base-10**. Computers use **Base-2** or **Binary**. Any electronic device using a computer runs on the binary system where electricity either flows or it does not flow. Information is processed and stored in the computer by a series of electronic switches. These switches are either on or off. If the switch is on **(1)**, electrical current is running through it. If the switch is off **(0)**, electrical current is **not** running through it. Every number, letter or special character that you can type from the keyboard is coded as a series of ones and zeroes; on and off signals.

Bit

In the computer, **one** on-off signal or switch is called a **bit**.

Byte

8 bits is a **byte.** A **byte** is a group of 8 switches; each turned either on or off. Each letter, number, or symbol on your keyboard is characterized by a **byte**.

Kilobyte

1024 bytes is a Kilobyte. Computer people shorten that to **K**. **1K** is equivalent to 1000 bytes. If you have **256K** of cache memory; that means you could store **256,000** characters of information in cache.

Megabyte

One million bytes is a **Megabyte** or **MB**. If a picture or video file is **6MB**, that file is 6 million bytes.

Gigabyte

One billion bytes is a **Gigabyte** or **G**. If you have an **8G** Flash Drive, you could store 80 billion characters of information on that Flash Drive.

Terabyte

One thousand Gigabytes is a **Terabyte** or **TB. =** 1000000000000 bytes = 10^{12} bytes

Petabyte

One Thousand Terabytes is a **Petabyte** or **PB** = 1000000000000000 bytes = 10^{15}bytes

And so on…

Since computer language, binary numbers, bits and bytes are all relative learning technology; let's look at the numbering systems.

Base-10 - to the Power of 10

The numbering system we conventionally use is based on the number 10. Look at the number 235; the 2 represents two hundreds, 3 represents three tens, and 5 represents five ones.

Look at the table below to see the relationship:

1000 or 10^3 (10 x 10 x 10)	**100** or 10^2 (10 x 10)	**10** or 10^1	**1** or 10^0
	2	3	5
200 + 30 + 5 = 235			

Binary - Base-2 - to the Power of 2

Based on the number **two** and called the **Binary Numbering System;** this numbering system is used for computers, including development of computer programs, creating files, measuring speed and storing information. Let's look at that same number **(235)** written in Binary.

128 or 2^7 (2 x 2 x 2 x 2 x 2 x 2 x 2)	**64** or 2^6 (2 x 2 x 2 x 2 x 2 x 2)	**32** or 2^5 (2 x 2 x 2 x 2 x 2)	**16** or 2^4 (2 x 2 x 2 x 2)	**8** or 4^3 (2 x 2 x 2)	**4** or 2^2 (2 x 2)	**2** or 2^1	**1** or 2^0 (2 x)
1	1	1	0	1	0	1	1
128 + **64** + **32** + **8** + **2** + **1** = **235**							

Converting Numbers

Using these two numbering systems; **Base-10** and **Binary**, let's look at various numbers and how you convert from one system to the other.

Base-10 Number = 23
Binary Number = 10111
Base-10 Number = 16 + 4 + 2 + 1 = 23

128 or 2^7 (2 x 2 x 2 x 2 x 2 x 2 x 2)	**64** or 2^6 (2 x 2 x 2 x 2 x 2 x 2)	**32** or 2^5 (2 x 2 x 2 x 2 x 2)	**16** or 2^4 (2 x 2 x 2 x 2)	**8** or 4^3 (2 x 2 x 2)	**4** or 2^2 (2 x 2)	**2** or 2^1	**1** or 2^0 (2 x)
0	0	0	1	0	1	1	1

Base-10 Number = 83
Binary Number = 1010011

Base-10 Number = 64 + 16 + 2 + 1 = 83

128 or 2^7 (2 x 2 x 2 x 2 x 2 x 2 x 2)	**64** or 2^6 (2 x 2 x 2 x 2 x 2 x 2)	**32** or 2^5 (2 x 2 x 2 x 2 x 2)	**16** or 2^4 (2 x 2 x 2 x 2)	**8** or 4^3 (2 x 2 x 2)	**4** or 2^2 (2 x 2)	**2** or 2^1	**1** or 2^0 (2 x)
0	1	0	1	0	0	1	1

Looking at some more examples:

Base-10 Number = 168
Binary Number = 10101000

128 or 2^7 (2 x 2 x 2 x 2 x 2 x 2 x 2)	**64** or 2^6 (2 x 2 x 2 x 2 x 2 x 2)	**32** or 2^5 (2 x 2 x 2 x 2 x 2)	**16** or 2^4 (2 x 2 x 2 x 2)	**8** or 4^3 (2 x 2 x 2)	**4** or 2^2 (2 x 2)	**2** or 2^1	**1** or 2^0 (2 x)
1	0	1	0	1	0	0	0

Binary Number = 11111
Base-10 Number = 16 + 4 + 2 + 1 = 31

128 or 2^7 (2 x 2 x 2 x 2 x 2 x 2 x 2)	**64** or 2^6 (2 x 2 x 2 x 2 x 2 x 2)	**32** or 2^5 (2 x 2 x 2 x 2 x 2)	**16** or 2^4 (2 x 2 x 2 x 2)	**8** or 4^3 (2 x 2 x 2)	**4** or 2^2 (2 x 2)	**2** or 2^1	**1** or 2^0 (2 x)
0	0	0	1	1	1	1	1

Adding in Binary

Add Base-10

Adding binary numbers together is very much like adding Base-10 numbers. Below we add 367 and 18 = 385. Add the ones column, if we have more than 10 we carry it over to the tens column. Then we add the tens column and if we have more than ten we carry it over to the hundreds column.

Base-10
367
+ 18
385

When you ask the computer to add 8 plus 4; it will return this:

Base-10	**Binary**
8	1000
	100
4 12	1100

When you ask the computer to add 8 plus 7; it will return this:

Base-10	**Binary**
8	1000
	111
7 15	1111

To demonstrate how binary carries over the same as Base-10 we will look at 11 plus 10. Remember; the same as Base-10, you must start on the right column.

Base-10 Number	**64** or 2^6 (2 x 2 x 2 x 2 x 2 x 2)	**32** or 2^5 (2 x 2 x 2 x 2 x 2)	**16** or 2^4 (2 x 2 x 2 x 2)	**8** or 4^3 (2 x 2 x 2)	**4** or 2^2 (2 x 2)	**2** or 2^1	**1** or 2^0 (2 x)
11 (8+2+1)	0	0	0	1	0	1	1
10 (8+2+0)	0	0	0	1	0	1	0
21(16+4+1)	0	0	**1**(0+0+1) ****	**0**(1+1) ***	**1**(0+0+1) **	**0**(1+1) *	**1**(1 +0)

1. Starting in the right **1's** column 1 + 0 = **1**
2. The **2's** column * 1 + 1 = 2 - we have to enter **0** and carry over the **1**
3. The **4's** column ** 0 + 0 + 1 = **1** - zero plus zero = zero, but we had **1** carried over
4. The **8's** column *** 1 + 1 = 2 - we have to enter **0** and carry over the **1**
5. The **16's** column **** 0 + 0 + 1 = **1** - zero plus zero = zero, but we had **1** carried over 6.

So the answer is **10101 = 21**

When you ask the computer to add 17 plus 20; it will return this:

Base-10 Number	**64** or 2^6 (2 x 2 x 2 x 2 x 2 x 2)	**32** or 2^5 (2 x 2 x 2 x 2 x 2)	**16** or 2^4 (2 x 2 x 2 x 2)	**8** or 4^3 (2 x 2 x 2)	**4** or 2^2 (2 x 2)	**2** or 2^1	**1** or 2^0 (2 x)
17 (16 + 1)	0	0	1	0	0	0	1
20 (16 + 4)	0	0	1	0	1	0	0
37 (32 + 4 + 1)	0	**1**	**0**(1+1) ****	**0**(0+0) ***	**1**(0+1) **	**0**(0+0) *	**1**(1+0)

1. Starting in the right **1's** column -- 1 + 0 = **1**
2. The **2's** column -- * 0 + 0 = 0
3. The **4's** column -- ** 0 + 1 = **1**
4. The **8's** column -- *** 0 + 0 = 0
5. The **16's** column -- **** 1 + 1 = **2** – so we enter **0** and carry over the 1
6. The **32's** column -- **** 0 + 0 = 0, but we had **1** carried over
7. So the answer is **100101**

Base-10	**Binary**
17	10001
20	10100
37	100101

Subtracting in Binary

Review Subtracting in Base-10

Subtracting 18 from 367 we know eight is larger than 7, so we cannot subtract them. We borrow from the 10s column. We borrow one ten and add it to 7. This makes 17. We can then subtract 8 from 17 to get 9. We borrowed one ten, instead of 6 tens we only have five. Five minus 1 equals 4. Three minus 0 equals 3.

Base-10
367
- 18
349

When we ask the computer to subtract; it works the problems just as we would in Base-10.
We ask the computer to subtract 2 from 14 and store the answer.

Base-10 Number	**64** or 2^6 (2 x 2 x 2 x 2 x 2 x 2)	**32** or 2^5 (2 x 2 x 2 x 2 x 2)	**16** or 2^4 (2 x 2 x 2 x 2)	**8** or 4^3 (2 x 2 x 2)	**4** or 2^2 (2 x 2)	**2** or 2^1	**1** or 2^0 (2 x)
14 (8+4+2)	0	0	0	1	1	1	0
2 (0+0+2)	0	0	0	0	0	1	0
12 (8+4+0)	0	0	0	**1**(1-0) ***	**1**(1-0) **	**0**(1-1) *	**0**(0-0)

1. Starting in the right **1's** column -- 0 - 0 = **0**
2. The **2's** column -- * 1 - 1 = **0**
3. The **4's** column -- ** 1 - 0 = **1**
4. The **8's** column -- *** 1 - 0 = **1**
5. So the answer is **1100**

When the top number is smaller than the bottom, we borrow, just as we would in Base-10.
We ask the computer to subtract 18 from 33 and store the answer.

Base-10 Number	**64** or 2^6 (2 x 2 x 2 x 2 x 2 x 2)	**32** or 2^5 (2 x 2 x 2 x 2 x 2)	**16** or 2^4 (2 x 2 x 2 x 2)	**8** or 4^3 (2 x 2 x 2)	**4** or 2^2 (2 x 2)	**2** or 2^1	**1** or 2^0 (2 x)
30 (16+8+4+2)	0	0	1	1	1	1	0
18 (16+0+0+2)	0	0	1	0	0	1	0
12 (0+8+4+0)	0	0	**0**(1-1) ****	**1**(1-0) ***	**1**(1-0) **	**0**(1-1) *	**0**(0-0)

1. Starting in the right **1's** column -- 0 - 0 = **0**
2. The **2's** column -- * 1 - 1 = **0**
3. The **4's** column -- ** 1 - 0 = **1**
4. The **8's** column -- *** 1 - 0 = **1**
5. The **16's** column -- **** 1 - 1 = **0**
6. So the answer is **1100**